CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

MATH 223: MECHANICS I

STREAMS: BSC (MATHS) **TIME: 2 HOURS**

DAY/DATE: MONDAY 03/12/2018 **11.30 AM – 1.30 PM**

INSTRUCTIONS:

- **Answer Question One and any other two Questions**
- **Adhere to the instructions on the answer booklet**

QUESTION ONE (30 MARKS)

(a) A moving box of a mass of 20kgs has a force of 250N applied to it at an angle of 30° . The box moves 16m in the horizontal direction. Calculate the work done by the force over the distance moved. [3 marks]

(b) If 20J of work is done in raising a 150g apple. How far is the apple lifted? [3 marks]

(c) A 750 kg object is pushed a long horizontal surface. After the object is pushed a distance of 45 m. starting from rest its speed is 6m/s. find the magnitude of the net force on the object. [3 marks]

(d) The velocity of a hockey ball before and after being struck by a stick are $\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -10 \\ 3 \end{pmatrix}$ m/s respectively. If the ball weighs 0.2kg and the contact last for 0.04 seconds.

Calculate the magnitude of the average force between the stick and the ball. [4 marks]

(e) A crane lifts a 25×10^2 kg beam vertically upwards with a force of $(26+4s^2)\,KM$.
Determine the speed of the beam, when it has risen 3m and the time it takes to attain this heights starting from rest. $g=9.81\,m\,s^{-2}$ [5 marks]

(f) A butcher throws a cut of beef on spring scales which oscillates about the equilibrium position with a period T=0.5s. The amplitude of the vibration A=2.0cm. calculate

(i) The frequency [1 marks]
(ii) Maximum acceleration [3 marks]
(iii) Maximum velocity [2 marks]

(g) Evaluate the angular momentum of a particle with position vector

$\vec{r}=(x,y)\wedge\vec{p}(p_x,p_y)$ [3 marks]

(h) Find the acceleration due to gravity for an airplane flying at an altitude of 20km. Given that

$= \; 6.67 \times 10^{-11} m^3/kg/s^2 = G$

Mearth= $5.98 \times 10^{24} kg$

Rearth = $6.37 \times 10^6 m$ [3 marks]

QUESTION TWO

(a) The end of alight inextensible string l metres long is fixed and a particle of mass m kg is attached to the other end. The particle is released from rest when the string is staut and horizontal.

(i) Show that the speed of the particle when the string makes angle θ to the horizontal is given by $V=\sqrt{2gl\cos\left(\frac{\pi}{2}-\theta\right)}$

[5 marks]

(ii) Given that the string is 160 m, find the speed of the particle when the string makes an angle of 40 $^\circ$ to the horizontal and the maximum speed of the particle.
[4 marks]

(b) A stone is thrown horizontally over the edge of a vertical cliff with an initial velocity of 30m/s. The cliff is 100 m high. Find the distance from the cliff at which the stone hits the ground and the magnitude of the velocity at this instant. [6 marks]

(c) A footballer kicks a ball towards the goal 20 m away with a velocity of $\begin{pmatrix} 30 \\ 10 \end{pmatrix} ms^{-1}$. The cross bar is 2m above the ground. Determine whether the footballer scores neglecting the effect of the goalkeeper. [5 marks]

QUESTION THREE

(a) The 200 kg block shown below rest on a smooth incline. If the spring is originally stretched through 1m, determine the total work done by all the forces acting on the block when force of 500N pushes the block up the plane through a distance of 3m. take the spring constant $K=30\,N\,m^{-1} \wedge g=9.81\,m\,s^{-2}$ [7 marks]

(b) A particle moving in a straight line with constant acceleration travels 110m in the first second, and 115 m in the 3rd second. Find

(i) Its initial velocity [2 marks]

(ii) Its acceleration [3 marks]

(iii) Distance traveled in the 2nd second [1 mark]

(c) Calculate the velocity of the 50kg crate shown below when it reaches the bottom of the cliff of B. if it is given an initial velocity of 4m/s down the cliff at A. the coefficient of kinetic friction, $\mu=0.3$. Distance from A to B is 20m. [6 marks]

QUESTION FOUR

(a) It is known that a load with mass of 20g will stretch a spring 1 cm. the spring is then stretches an additional 0.5 cm, and released. Find

(i) The spring constant [2 marks]
(ii) Period of vibration & frequency [2 marks]
(iii) Maximum acceleration [2 marks]
(iv) The equation of motion [2 marks]

(b) A $2.5 \times 10^4 kg$ car accelerates uniformly from rest to 10 m/s in 4s.

(i) Find the work done on the car in this time interval [2 marks]

(ii) Find the power delivered by the engine in this time interval. [2 marks]

(c) A pendulum bob is released from some initial height such that the speed of the bob at the bottom of the swing is 2.5 m/s. Find the initial height of the bob. [3 marks]

(d) A machine mounted on vibration isolators is modeled as a single degree of freedom system. The mass m=370 kg, spring rate $k = 2 \times 10^5 \mu/m,$ damping constant $\delta = 0.2/s.$ Calculate the natural frequency of the mounted machine and the displacement amplitude of the machine if it is excited at that frequency by a force with a peak amplitude of 10N. [5 marks]

QUESTION FIVE

(a) A spaceship is between the earth and moon. Find the distance from the earth at which the net gravitational force on the spaceship is zero. [8 marks]

(b) Find the gravitational force due to a hollowed sphere, assuming that the mass of the sphere was M before hollowing. [5 marks]

(c) Find the radius of the orbit of a geosynchronous satellite given that the period of the orbit is 86400s. [4 marks]

(d) Find the acceleration due to gravity for a geosynchronous satellite at 3600 km. [3 marks]
