CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

ACMT 401: LIFE CONTINGENCIES II

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: THURSDAY 31/08/2023 **8.30 A.M – 10.30 A.M.**

INSTRUCTIONS

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

(a) State four transitions that, in real life, might affect the expected present value of thesickness benefits for a new policyholder who is currently healthy. Assume that the policyholder is to pay regular premiums.
(4 Marks)

(b) Calculate the following using ELT15 (Males) (6 Marks)

i. $_{5|2}q^{1}_{64:64}$

ii. $_{13}p_{\overline{45:45}}$

(c) Jared, aged 60, and Joyce, aged 65, took out a life assurance policy that provides a payment of £10,000 immediately when the second of them dies. Level annual premiums are payable in advance whilst the policy is in force. Calculate the annual gross premium, using the basis given below:
(5 Marks)

Basis:

Mortality: PA92C20
Interest: 4% pa effective
Expenses: Initial: £ 75
Renewal: 3% of each premium, excluding the first

(d) A life insurance company uses the three-state healthy-sick-dead model described above to calculate premiums for a 3-year sickness policy issued to healthy policyholders aged 60.

Let St denote the state occupied by the policyholder at age 60 + t, so that S0 = H and St = H, S or D for t = 1,2,3.
The transition probabilities used by the insurer are defined in the following way:

$$p_{60+t}^{jk} = P\left(S_{t+1} = k \middle| S_t = j\right)$$

For t = 0,1,2, it is assumed that:

$$p_{60+t}^{HH} = 0.9 \qquad p_{60+t}^{HS} = 0.08$$
$$p_{60+t}^{SH} = 0.7 \qquad p_{60+t}^{SS} = 0.25$$

Calculate the probability that a new policyholder is:

(i) sick at exact age 62 (2 Marks)

(ii) dead at exact age 62 (3 Marks)

(e) A life office sells joint whole life assurances to male lives aged 63 and female lives aged 68 exact. The benefits, payable at the end of the year of death in each case, are £120,000 on the first death and £65,000 on the second death. Level premiums are paid annually in advance while the policy is in force. Calculate:

(i) the annual premium payable (5 Marks)

(ii) the prospective reserve just before the payment of the 7th annual premium, assuming both lives are still alive at that point. (5 Marks)

QUESTION TWO (20 MARKS)

(a) Discuss the actuarial control cycle in the management of financial risks. (5 Marks)

(b) Express each of the following symbols. (6 Marks)

(i) ${}_{\infty}q_{xy}^{1}$

(ii) $\bar{A}_{xy}^{1}$

(iii) $\bar{A}_{x\bar{y}}^{2}$

(c) Given that:

$$\mu_x = \frac{1}{100-x} \quad \text{for} \quad 0 \le x < 100$$

calculate the value of ${}_{30}q_{50:60}^{2}$. (5 Marks)

(d) Two lives aged x and y take out a policy that will pay £15,000 immediately on the death of (x) provided that (y) has died at least 5 years earlier and no more than 15 years earlier.

i. Express the present value of this benefit in terms of the random variables denoting the future lifetimes of (x) and (y). (2 Marks)

ii. Write down an integral expression (in terms of single integrals only) for the expected present value of the benefit. (2 Marks)

QUESTION THREE (20 MARKS)

(a) A company provides the following benefits for its employees:

- immediately on death in service, a lump sum of Kshs.18,000
- immediately on withdrawal from service (other than on death or in ill-health), a lump sum equal to Kshs. 2,000 for each completed year of service
- immediately on leaving due to ill-health, a benefit of Kshs. 8,000 p.a payable monthly in advance for 5 years certain and then ceasing, and
- on survival in service to age 53, a pension of Kshs. 3,500 pa for each complete year of service, payable monthly in advance from age 53 for 5 years certain and life thereafter.

The forces of decrement for the employees at each age, assumed to be constant over each year of age,are as follows:

Age, x	$\mu_{\bar{x}}^{d}$	$\mu_{\bar{x}}^{i}$	$\mu_{\bar{x}}^{w}$
50	0.016	0.10	0.020
51	0.020	0.15	0.015
52	0.024	0.22	0.010

where $\mu_{\bar{x}}^{j}$ is the (assumed constant) force of decrement by cause j over (x, x+i), *d* represents death, *i*represents ill-health retirement and *w* represents withdrawal.

i. Construct a multiple decrement table with radix $(al)_{50}$= 100,000 to show the numbers of deaths, ill-health retirements and withdrawals at ages 50, 51 and 52, and the number remaining in employment until age 53. (5 Marks)

ii. Calculate the expected present value of each of the above benefits for a new entrant aged exactly 50. Assume that interest is 6% pa effective before retirement and 4% pa effective thereafter, and that mortality after retirement follows the PMA92C20 table. (10 Marks)

(b) A population of healthy people over the year of age 56 to 57 is subject to a constant force of decrement due to sickness of 0.07 per annum, and a constant force of mortality of 0.003 per annum.

Assuming that a double decrement model is used, calculate:

i. the probability that a healthy person aged exactly 56 will still be healthy at exact age 57. (1 Mark)

ii. the probability that a healthy person aged exactly 56 will leave the healthy population through death before exact age 57.
(2 Marks)

iii. the independent probability of a life aged exactly 56 dying before exact age 57.
(2 Marks)

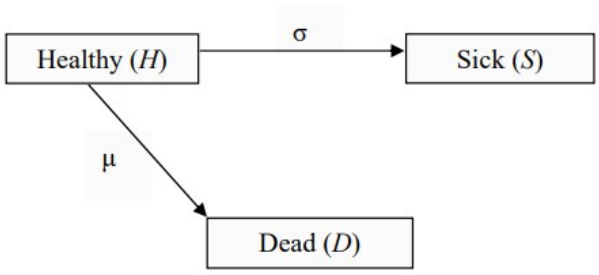
QUESTION FOUR (20 MARKS)

(a) The gross future loss random variable at inception for a policy issued to a life, aged 50 exact, is given by:

$$1{,}500a_{\overline{\min(K_{50},\,15)}|} + 35\left(\ddot{a}_{\overline{\min(K_{50}+1,\,15)}|} - 1\right) + 200 - 0.97 \times P$$

Where:
K_x = curtate future lifetime of a person aged x exact
P = Premium paid.
Describe in words the timings and amounts of the benefits, expenses and premium of the policy implied by the gross future loss random variable.
(7 Marks)

(b) A life insurance company sells a special 15-year pure endowment assurance policy, where a sum assured of $10,000 is paid at the end of the term if the policyholder remains healthy throughout.
The following three-state transition model is used:



i. Show that the expected present value of the benefit, in respect of a healthy life aged 50 exact, is approximately equal to $1,054.
(6 Marks)
Premiums of P per annum are payable continuously throughout the policy term, ceasing on death or if the policyholder becomes sick.

ii. Calculate P, showing all working. (7 Marks)

Basis:

$\mu = 0.04$
$\sigma = 0.08$
Force of interest: 3% per annum

QUESTION FIVE (20 MARKS)

(a) Differentiate between stochastic and deterministic models.
(2 Marks)

(c) (i) Explain what it means for the last survivor status $\overline{45:50}$ to remain active for at least 10 years. (2 Marks)

(ii) Calculate the probability that the event described in part (i) occurs, assuming the twolives areindependent with respect to mortality and:

(a) the mortality of each life follows the ELT15 (Females) table. (4 Marks)

(b) each life is subject to a constant force of mortality of 0.015 pa. (2 Marks)

(b) A life insurance company issues a retirement policy to a male policyholder aged 60 exact. The policy provides the following benefits:

- An immediate whole life level annuity of £15,000 per annum payable monthly in advance.
- A reversionary annuity of £5,000 per annum payable monthly in advance to the policyholder's surviving female spouse, who is currently aged 62 exact.
- A lump sum of £200,000 payable immediately to the policyholder's family if the policyholder dies before age 65.

Calculate the expected present value of the policy benefits at outset. (10 Marks)

Basis:	
Mortality	Male PMA92C20
	Female PFA92C20
Interest	4% per annum effective
