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MATHEMATICAL MODELING OF STERILE INSECT TECHNIQUE FOR CONTROL OF FALSE CODLING MOTH

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ABSTRACT
The Sterile Insect Technique (SIT) is a biological and nonpolluting method of control of pest population in a farm. This method relies on the release of sterile male False Codling Moth (FCM) in order to reduce the population fertile female FCM in the farm. In this study a mathematical model that simulates the interaction between the susceptible host, sterile male FCM and the wild FCM population is developed. The local and global stability analysis of the model is analyzed and found to be asymptotically stable when $R_o \leq 1$. A threshold number of sterile insect FCM above which the control of FCM is effective is determined. The results are reorganized as possible strategies for control of FCM and illustrated.
Keywords: Sterile insect technique, False codling moth, Stability analysis

INTRODUCTION

For more than half a century, the Sterile Insect Technique (SIT) has been used successfully to eliminate or control a variety of pest species, mostly dipteran (Barclay, 2016). Knipling (1955) introduced this method, which entails releasing large numbers of sterilized males into the environment. This technique is a biological control method that prevents insect pests from reproducing naturally. This is achieved by treating male insect pests with chemical, physical, or other radical procedures to render them infertile, preventing them from reproducing regardless of their sex drive (Ben Dhahbi *et al*., 2020).

In SIT, the sterile organisms mate with fertile organisms leading to unfruitful mating, thereby, controlling their population by reducing the number of viable offspring (Dufourd and Dumont, 2012). Consequently, leading to decline of FCM population with time. Therefore, releasing sufficiently many sterile males into the wild FCM population over a sufficiently long period of time can lead to reduction or elimination of the FCM in a farm (Peshin & Dhawan, 2009; Anguelov *et al.*, 2012). This technique has given farmers the ability to control certain insects' pest in livestock, fruits, vegetables, and fibre crops. It is also an environmentally friendly approach, as it uses no noxious chemicals, leaves no residues, and 3 is entirely species specific, thus having no non- target effects on crops and livestock (Dyke *et al*., 2006). However, in most SIT model non has been used in the control of False Codling Moth (FCM). The main critical factors in pest control using SIT is the critical sterile release rate, and the over flooding ratio, which is the measure of the ratio of sterile males to wild males (Steiner et al., 1965).

False codling moth (FCM), *Thaumatotibia leucotreta* is considered the most significant indigenous pest (Gillaga *et al*., 2011). This is due to its potential economic impact on many horticultural and agricultural crops, with crop losses ranging from 20−90% (Savary, 2006; Gillaga *et al.*, 2011). Consequently, FCM is a major threat to food security, supply of raw material for manufacturing, foreign exchange and employment in many countries (Blomefield *et al*., 1989; Gillaga *et al*., 2011).

Sterile release models were developed by Anguelove *et al,* (2012) to control anopheles' mosquito and Barclay (2016) to determine the rate of sterile release. Most of these mathematical models do not address the population dynamicsof FCM and the control measure in which this study is seeking to address.

Planning efficient and cost effective control is a real challenge which can explain the failure of most experimental methods of FCM control strategy (Anguelove *et al.*, 2016). This is because biological systems can be made unstable or stable if certain parameters are changed appropriately, that is, if their values pass through bifurcation values (Murray, 2002; Sergio, 2014; Savary, 2006). Therefore, there is need for more scientific studies on how changes such as SIT and pheromone trap affect the biological system of host-pest relationships.

Carpenter et al. (2014) conducted a study on area- wide control tactics for the false codling moth Thaumatotiba leucotreta by integrating sterile release technique with the release of the egg parasitoid. In the study, both treated males and females were released in the field together with parasitoid Trichogramma cryptophlebia. The result showed that combined field release of irradiated false codling moth and parasitoid resulted in rapid parasitoid population increase, which had a positive impact on false codling moth population suppression that is more effective than when either techniques were employed separately. Therefore, FCM is a pest of phytosanitary concern and it impedes export in most international markets, as it is endemic to sub-Saharan Africa (Hofmeyr *et al*., 1998; Moore, 2012). Consequently, interception of even an individual pest in a consignment could lead to rejection of the entire consignment (Moore, 2012; Mkiga et al., 2019).

Most of the farmers rely on insecticide to reduce economic loss caused by FCM (Kalajdzievska and Li, 2007). Chemical pesticides have been extensively used to control pest population such as FCM for a long time all over the world. However, their extensive use has led to environmental pollution and reduction of pest's natural enemies resulting to undesirable side effects on the environment (Carde, 1990). Moreover, the development of insect resistance to the chemical products has also led to the rising need of stronger and more toxic pesticides to maintain their efficacy. Extensive use of pesticides is not a sustainable solution for pest control (Anguelov et al., 2016). As a result, there is growing interest on development of control strategies that do not pollute the environment, these strategies have a special focus on the ecology and behavior of the involved species. Therefore, to prevent devastating impact on economy (Wilson et al., 2015), and ensure sufficient food security, social life, health and biodiversity; efficient control, understanding of biodiversity and management of FCM is essential (Anguelove et al., 2016; Stibick, 2008). One of the most promising strategy is the use of sterile insect technique (SIT) and Pheromone traps (Roumen et al., 2010). In this paper, we study the host pest interaction in the present of sterile insect technique focusing on the false codling moth.

Model Formulation

The model is divided into three main compartment: susceptible host compartment (S_h), pest compartment (False codling moth compartment) and sterile insect compartment (M_s). The pest compartment if further divided into fertile egg compartment (E_f), fertile larvae compartment (L_f), fertile pupae compartment (P_f), fertile female compartment (F_f), fertile male (M_f), fertilized fertile female compartment (F_{ff}) and non-fertilized fertile female compartment (F_{nf})

We consider a two-dimensional multi-stage model that involves density dependence solely in the growth term of FCM insects and the susceptible host. We assume that the birth rate of the sterile insects is their release rate and that the sterile-fertile competition affects only the wild population.

Let $S_h(t)$ be the number of susceptible host at time t. In the absence of FCM it evolves according to the following equation

$$\frac{dS_h}{dt} = \alpha\left(1 - \frac{S_h}{K_h}\right)S_h - \mu_1 S_h, \quad t \geq 0 \tag{1}$$

Where μ_1 is the harvesting rate and K_h is the environmental carrying capacity, α is the intrinsic growth rate of the susceptible host? Since FCM attack the susceptible host at the larval stage, by introducing the FCM attack rate to the susceptible host. Equation 1 can be rewritten as:

$$\frac{dS_h}{dt} = \alpha\left(1 - \frac{S_h}{K_h}\right)S_h - \frac{\xi S_h L_f}{m + S_h} - \mu_1 S_h, \quad t \geq 0 \tag{2}$$

Where ξ is the attack rate by larvae, m is the half saturation constant and L_f is the number of larva presents at time t.?

It is important to note that at larval stage the FCM is most destructive to the susceptible host. The sterile insect directly affects the fertile male female compartment and we now proceed to formulate the equation of FCM stages as follows. Let the E_f be the number of eggs laid by the fertilized fertile female after successful mating with the fertile male at time t

$$\frac{dE_f}{dt} = rF_{ff}\left(1 - \frac{E_f}{A}\right)\phi S_h - (\lambda_1 + \omega_1)E_f \tag{3}$$

Where r is the intrinsic egg laying rate, A is the carrying capacity of fertile egg, ϕ is the egg conversion rate, λ_1 is transfer rate from fertile eggs stage to fertile larva and ω_1 is the natural egg mortality rate.

The larval stage of FCM can be modelled using the following equation:

$$\frac{dL_f}{dt} = \lambda_1 E_f + \frac{a\xi S_h L_f}{m + S_h} - (\lambda_2 + \omega_2)L_f \qquad (4)$$

Where a is the larval conversion rate and λ_2 is the transfer rate from larval stage to pupal stage and ω_2 is the natural larval mortality rate.

At pupal stage the equation describing the evolution of pupal stage over time is given by

$$\frac{dP_f}{dt} = \lambda_2 L_f - (\lambda_3 + \omega_3)P_f \qquad (5)$$

After emergence from pupal stage FCM can either be fertile female or fertile male. The equation of the fertile female is given by

$$\frac{dF_f}{dt} = \kappa\lambda_3 P_f + \delta_1 F_{ff} + \delta_2 F_{nf} - \left(\lambda_4\left(\frac{M_f}{M_f + M_s}\right) + \omega_4\right)F_f \qquad (6)$$

Where κ is the fraction of pupal population that emerged into fertile female, δ_1 fraction of fertilized fertile female that returned to the fertile female after laying fertile eggs, δ_2 fraction of fertilized fertile female that returned to the fertile female after laying non-hatching eggs after mating with the sterile insect λ_4 is the transfer rate of the fertile female either fertilized fertile female or non-fertilized fertile female and ω_4 is the natural mortality rate of fertile female.

The equation that describe the fertile male is given by

$$\frac{dM_f}{dt} = (1 - \kappa)\lambda_3 P_f - \omega_3 M_f \qquad (7)$$

Where ω_3 is the natural mortality of the fertile male? If the sterile insect is released at the rate of ψ and q is the fraction of the released sterile male that join the wild FCM population and ω_8 is the natural mortality of sterile insect.

If the μ is the mating competitiveness of the sterile male, then the equation governing the sterile insect compartment is given by

$$\frac{dM_s}{dt} = \psi q\mu - \omega_8 M_s \qquad (8)$$

Let the fraction of fertile female that mate with the fertile male and move to the fertilized fertile female compartment to lay a fertile egg be given by $\left(\frac{M_f}{M_f + M_s}\right)$ and $\left(\frac{M_s}{M_f + M_s}\right)$ to represent fraction of fertile female that mate with the sterile male and move to the non-fertilized female compartment and lay a non-hatching eggs. Then the equation that describe the evolution of fertilized fertile female and non-fertilized fertile female is given by

$$\frac{dF_{ff}}{dt} = \left(\lambda_4\left(\frac{M_f}{M_f + M_s}\right)\right)F_f - (\delta_1 + \omega_6)F_{ff} \qquad (9)$$

$$\frac{dF_{nf}}{dt} = \left(\lambda_4\left(\frac{M_s}{M_f + M_s}\right)\right)F_f - (\delta_2 + \omega_7)F_{ff} \qquad (10)$$

Where ω_6 natural mortality is fertilized fertile female and ω_7 is the natural mortality of non-fertilized fertile female.

Taking into account the above considerations, we have the schematic flow diagram shown in Figure 1. From Figure 1, the model will be governed by the following equations

$$\frac{dS_h}{dt} = \alpha\left(1 - \frac{S_h}{K_h}\right)S_h - \frac{\xi S_h L_f}{m + S_h} - \mu_1 S_h,$$

$$\frac{dE_f}{dt} = rF_{ff}\left(1 - \frac{E_f}{A}\right)\phi S_h - (\lambda_1 + \omega_1)E_f$$

$$\frac{dL_f}{dt} = \lambda_1 E_f + \frac{a\xi S_h L_f}{m + S_h} - (\lambda_2 + \omega_2)L_f$$

$$\frac{dP_f}{dt} = \lambda_2 L_f - (\lambda_3 + \omega_3)P_f$$
$$\frac{dF_f}{dt} = \kappa\lambda_3 P_f + \delta_1 F_{ff} + \delta_2 F_{nf} - \left(\lambda_4\left(\frac{M_f}{M_f + M_s}\right) + \omega_4\right)F_f \tag{11}$$
$$\frac{dM_f}{dt} = (1-\kappa)\lambda_3 P_f - \omega_3 M_f$$
$$\frac{dM_s}{dt} = \psi q\mu - \omega_8 M_s$$
$$\frac{dF_{ff}}{dt} = \left(\lambda_4\left(\frac{M_f}{M_f+M_s}\right)\right)F_f - (\delta_1 + \omega_6)F_{ff}$$
$$\frac{dF_{nf}}{dt} = \left(\lambda_4\left(\frac{M_s}{M_f + M_s}\right)\right)F_f - (\delta_2 + \omega_7)F_{nf}$$

Model Analysis
All the feasible solutions of model system enter the region
$\Omega = \left\{(S_h(t), E_f(t), L_f(t), P_f(t), F_f(t), M_f(t), F_{ff}(t), F_{nf}(t), M_s(t)) \in R_+^9 : N(t) \le \frac{k}{q}(\alpha+1) + \xi\right\}$ for all $\xi > 0$ and $t \to 0$. Therefore, the system 11 is considered to be positively invariant and attracting and it is sufficient to consider solutions in. ΩThe existence, uniqueness and continuation results of system 1 hold in the region and all solutions starting in Ω remain in there for all $t \ge 0$. Hence system 11 is mathematically well posed and its sufficient to consider the dynamics the flow generated by the model system 11 in Ω. Also all parameter and state variables for the model system 11 in Ω are assumed to be non-negative since it monitors plants and pests populations.

Equilibria Analysis
In this study, we consider two main equilibrium point of interest: pest free equilibrium (E_0) and coexistence equilibrium (E_1). At Pest Free Equilibrium, it is assumed that there is no pest prevalence in the system hence the population of susceptible host grows logistically till harvesting time. All the pest compartment and sterile insect compartment of the system model 11 is set to zero. Therefore, solving the model system 11 the pest free equilibrium is given by:
$E_0 = (K_h(\alpha - \mu_1), 0, 0, 0, 0, 0, 0, 0, 0)$

Basic Reproduction Number
Using the method of van den Driessch and Watmough, (2002) the basic reproduction number of the model system 11 is obtained using next generation matrix method as:
$$R_o = \frac{\alpha\xi K_h(\alpha-\mu_1)}{(m+K_h(\alpha-\mu_1)(\lambda_2+\omega_7)} \tag{12}$$
In this study, the basic reproduction number is also referred to as the basic offspring number which is defined as the number of offspring produced by fertilized fertile female throughout its life time when placed in a completely susceptible host at its carrying capacity. The sterile insect affects the basic offspring number by reducing the number of fertilized fertile female population. If the basic offspring number is less than unity it means that the pest infestation would be reducing as less and less fertile eggs would be laid, but if basic offspring number is greater than unity it means that there is likely hood of possible outbreak of pest infestation.

Numerical Simulation
To understand the effect of sterile insect technique on the population of FCM and susceptible hosts numerical simulations of the system model 11 are carried out. The parameter values given in Table 1 were obtained based on data from literature about the biological control of false codling moth, and some of the values were assumed following realistic ecological observations. Numerical values are simulated at $0 \le t \le 100$ in days where most crops reproduction life cycle lies. The simulations are performed with the help of the MATLAB mathematical software and results are presented in graphical form. The impact of sterile insect on the host and FCM is illustrated in Figures 1, 2, 3 and 4. Figure 1 shows a plot of the number of sterile against time within a period of 10 days, 30 days and 100 days, with initial value of sterile insect set at zero and release rate $\psi = 100$. From the graph its observed that the number of sterile insect increase gradually up to a maximum value with time and follows the same trajectory at all times. Any further increase in time does not require additional increase in the sterile insect.

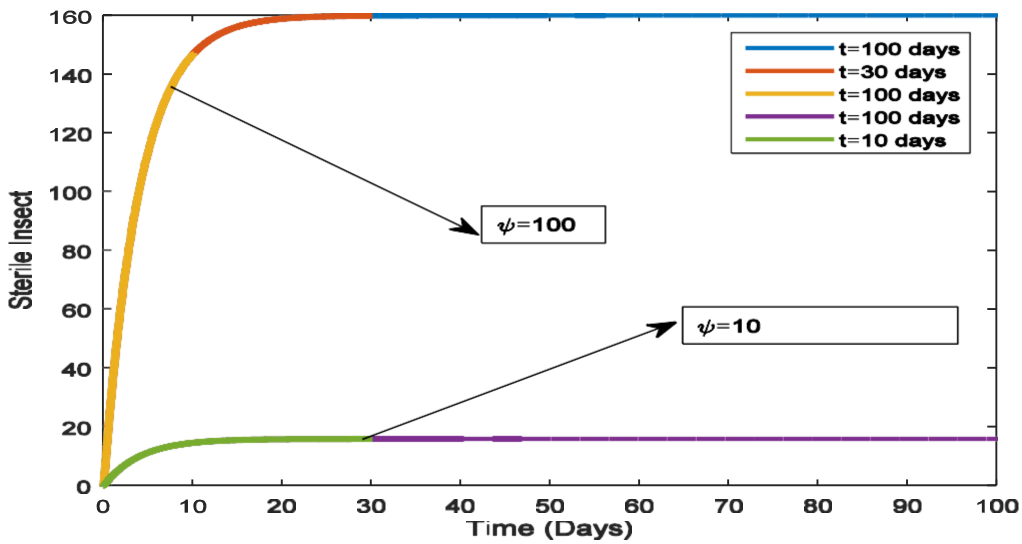


Figure 2 shows a plot of Susceptible host against sterile insect as a control measure of FCM, with the initial number of sterile insect set at zero and the release rate per day varied from 5, 10, and 100 sterile release per day. The susceptible host carrying capacity is set constant at K_h = 1000. From the graph it is evident that the minimum number of sterile release rate that would bring the susceptible host to its carrying capacity is approximately 5 sterile insects per day any further increase in the sterile insect would have no effect on the yields of susceptible host. This shows that sterile insect techniques help control the FCM population from the farm hence leading to high yields.

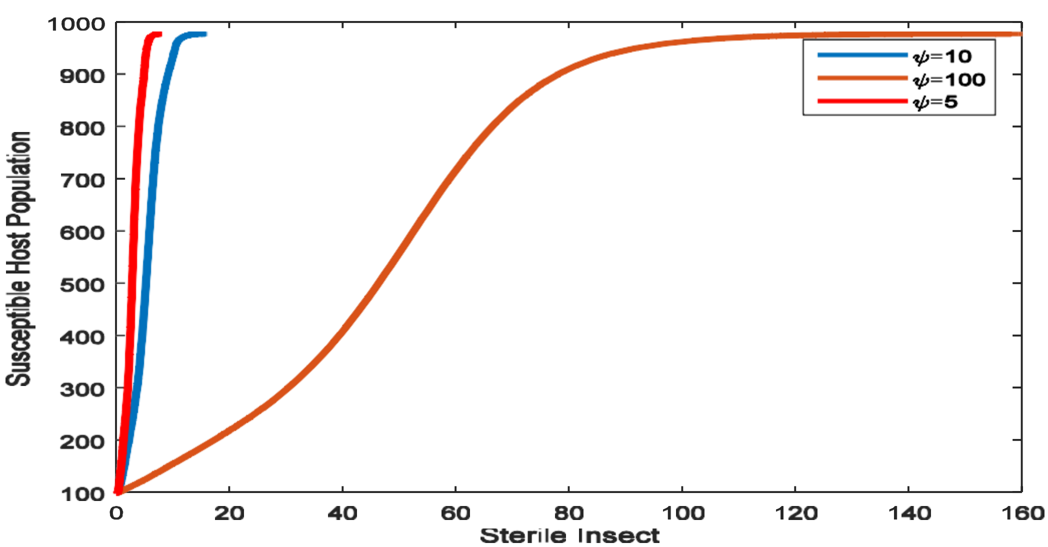


Figure 3 shows a plot of the total FCM population against the number of sterile insect released in the field under various release rate per day within a period of 30 days while keeping other parameters as indicated in Table 2 and 3. From the graph it is observed that the population of FCM first increases from the initial population to a maximum and gradually drops to zero. The 10 sterile insect release per day is found to be more effect than 100 sterile insect release per day. This shows the impact of sterile insect on the reduction of the population of FCM as a biological control measure.

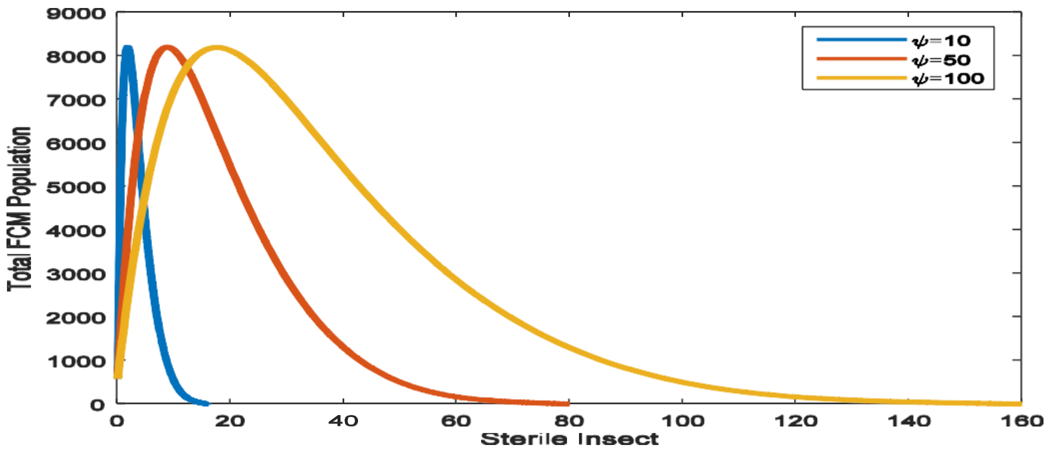


Figure 3 shows a plot of adult FCM population against the sterile insect, with the sterile release rate ψ = 100 within a period of 30 days. From the Figure the number of adult FCM decreases from the initial values of 100 in each compartment to 0. This indicates the effectiveness of sterile insect release as an FCM biological control method.

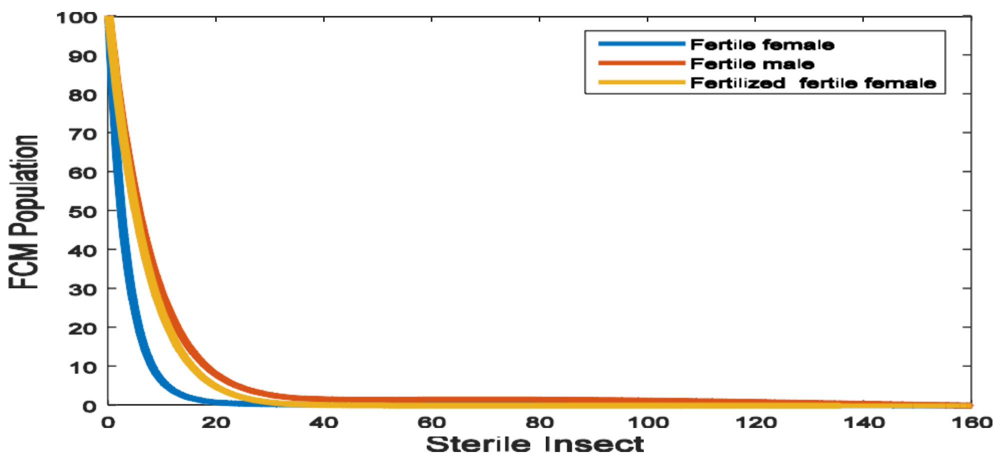
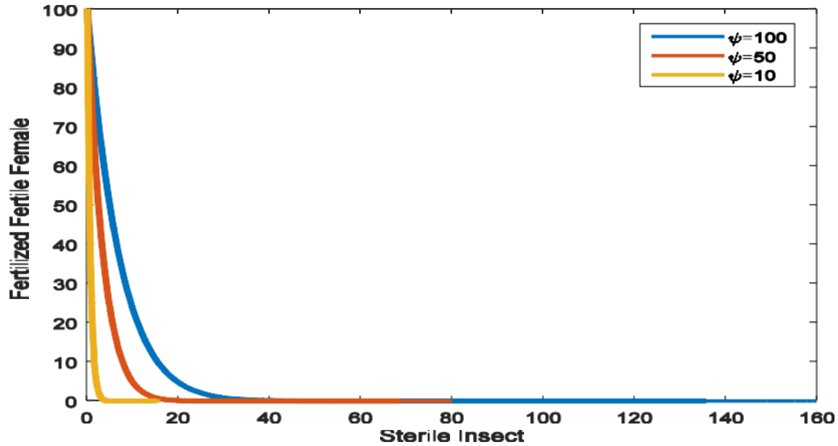


Figure 4 illustrate a plot of fertilized fertile female population against sterile insect under different release rates that is when ψ = 100, ψ = 50 and ψ = 10. From the Figure it is observed that the number of fertilized fertile female decreases from the initial population of 100 gradually to near zero with the increase in sterile insect and remain constant at near zero. Higher release rate does not provide a faster control as illustrated in Figure 10. Sterile insect therefore helps in reducing the number of fertilized fertile female by making the fertile female infertile through mating and only able to produce infertile eggs, hence helps in reducing the population of FCM.

DISCUSSION

Pest control has advanced significantly in recent years, owing to the realization that pesticides alone are frequently insufficient, if not counter-productive, for many pest problems, particularly on FCM. It is becoming increasingly clear that an adequate understanding of the biological dynamics associated with pest control techniques is critical in any pest control program.

A deterministic mathematical model of sterile insect technique for control of false codling moth is developed in this study. Although some researchers, such as Anguelove et al. (2012) and Anguelove et al. (2016), have developed biological pest control models, none of these models have taken the host compartment into account. Furthermore, most of these models have focused on general pest control rather than specific pest species, and some of the models have only captured pest dynamics while ignoring crop population, which has a significant impact on crop performance and pest interactions.



One of the most important aspects of pest control is keeping pest densities below economic thresholds. These levels are closely related to the crop population's dynamic evolution. As a result, when developing model system 11, we incorporated the dynamic of susceptible host population, FCM age structured compartments. We carefully examined the model and determined three ecological equilibrium points for the system model: I pest free equilibrium point (E0); ii) coexistence equilibrium point (E1); and iii) host free equilibrium point (E2). Because of its triviality, the host free equilibrium has not been subjected to stability analysis. The primary goal of this study is to determine the sterile release rate threshold parameter values that will cause FCM to extinction, thereby ensuring the survival of susceptible hosts.

CONCLUSION

From our analytical and numerical analysis of the system model, establishes a critical value of sterile insect release rate such that when these parameter values are crossed, the FCM will become extinct. We also validated this finding through numerical simulation. The analysis of the model showed that there exists a domain where the model is entomologically and mathematically well posed. Using Next generation matrix, the basic reproduction numbers of the system was computed. The numerical simulation of the model showed that sterile release are effective ways of controlling FCM population when a critical release value of sterile release rate is met.

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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