CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS, BACHELORS OF SCIENCE

MATH 400: TOPOLOGY I

STREAMS: B.ED (SCI & ARTS) AND BSC **TIME: 2 HOURS**

DAY/DATE: TUESDAY 04/12/2018 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS:

- **Answer Question ONE and ANY Other TWO Questions.**
- **Do not write on the question paper.**

QUESTION ONE: (30 MARKS)

(a) Define a topological space (X,τ). Hence find all possible topologies on X = {a, b}.

(5 marks)

(b) (i) Prove that if A is a subset of a discrete topology, then set of its derived points A' is empty.

(4 marks)

(ii)Prove that if x is a discrete topological space and $A=x$, then $\acute{A}=A$

(2 marks)

(c) Let $f: X \to Y$ be a constant function. Prove that then f is continuous relative to τ_X and τ_Y.

(3 marks)

(d) Using a counter example show that an open function or a closed function need not be continuous (3 marks)

(e) When is a subset A said to be dense in X ? Hence prove that if A is dense in X then for every open set subset $O\subset X$, O intersection A is non-empty. (4 marks)

(f) (i) Let $a\in R$. Show that every closed interval $[a-\delta,a+\delta]$ with the centre a is neighborhood of a. (2 marks)

(ii) Let $p\in X$ and denote N_P the set of all neighborhood of a point p. Prove that $\forall\, pairs\, N,M\in N_P, N\cap M\in N_P$ (3 marks)

(g) Distinguish a regular space and a normal space. Give one example in each case. (4 marks)

QUESTION TWO: (20 MARKS)

(a) Let (X,τ) be a topological space, prove that finite union of closed sets is also closed. (3 marks)

(b) Consider the discrete topology D on $X=\{a,b,c,d,e\}$. Find a subbase which does not contain any singleton sets (5 marks)

(c) If θ is a subbasis for the topologies τ and $\tau^{¿}$ on X, show that τ = $\tau^{¿}$

(5 marks)

(d) Let B be a subset of a topological space (X,τ). Prove that τ_B is a topology on B, where $\tau_B=\{B\cap G: G\in\tau\}$

(7 marks)

QUESTION THREE: (20 MARKS)

(a) Prove that a point $p\in X$ is an accumulation point of $A\subset X$ iff every member of some local base β_p at the point p contains a point of A different from

p. (5 marks)

(b) State and prove the Kuratowski's closure axioms of a topological space (X, τ).

(7 marks)

(c) Distinguish a T_1 space and a T_2 space, hence using appropriate counter examples show that a T_2 space $\Rightarrow T_1$ space and T_1 space $\Rightarrow T_0$ space but a T_0 space $\nRightarrow T_1$ and a T_1 space $\nRightarrow T_2$.

(8 marks)

QUESTION FOUR: (20 MARKS)

(a) Let $g: X \to Y$ be a bijective. Prove that the following statements are equivalent.

(i) g is a homomorphism

(ii) g is open

(iii) g is closed

(iv) $g(\acute{A}) = g(A)'$ (14 marks)

(b) Let $P: X \to Y$ be an open map and let $S \subset Y$ be any subset of Y and A is a closed set in X such that $P^{-1}(S) \subset A$. Show that $S \subset B$ and $P^{-1}(B) \subset A$.

(6 marks)

QUESTION FIVE: (20 MARKS)

(a) Let (X, τ) be a topological space. Prove that a subset $A \subset X$ is closed if and only if A contains all its limit points i.e. $A' \subset A$

(8 marks)

(b) Let β be a class of subsets of a non-empty set X. Prove that β is a base for some topology on X iff

(i) $X = \cup \{B : B \in \beta\}$

(ii) For any $B, B' \in \beta$, $B \cap B'$ is the union of members of β (12 marks)
