CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS 2023

FIRST YEAR EXAMINATIONS FOR THE AWARD OF BACHELOR OF SCIENCE IN MATHEMATICS.

MATH 326: METHODS OF APPLEID MATHS 1 TIME: 2 HOURS

INSTRUCTIONS

Answer question one and any other two questions

Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

a. Identify the nature of the singular points of the equation $4x^2y'' + 5xy' + \frac{1}{2}(x-1)y = 0$ 5mks

b. Find the series solutions about $x=0$ of $y'' + y = 0$ 6mks

c. Decompose $\frac{s+3}{(s-2)(s-3)}$ into partial fractions hence evaluate $L^{-1}\left(\frac{s+3}{(s-2)(s-3)}\right)$ 5mks

d. Find the sine Fourier series for the function $f(x) = 1,\ in\ 0 < x < \pi$ 5mks

e. Determine the nature of the singular points of the differential equation $\frac{3}{2}xy'' + y' + \frac{1}{2}y = 0$, Hence Find the roots of it's indicial equation 6mks

f. Evaluate the $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$ 5mks

QUESTION TWO

a. The Legendre`s equation has the form $\left(1-z^2\right)y'' - 2zy' + l\left(l+1\right)y = 0$, where l is a constant and z is the dependent variable,

i. Show that , $z=0$ is a an ordinary point and $z=\pm 1$ is a regular singular point of the equation 5mks

ii. Show that the Legendre`s equation has a regular singularity as $|z| = \infty$ 7mks

b. A periodic function f(t) of period 2π is defined by $f(t) = t^2 + t,\ -\pi < t < \pi$. Evaluate b_n, a_0 *and* a_n and obtain the Fourier series expansion of the function 8mks

QUESTION THREE

a. Using the Laplace transforms, to evaluate $\int_0^\infty te^{-3t}\sin t\,dt$ 5mks

b. Given the Bessel`s differential equation $x^2y'' + xy' + \left(x^2 - n^2\right)y = 0$, about the point x = 0.

i. Obtain the roots of the indicial equation of the differential equation 7mks

ii. Find the recurrence relation satisfied by coefficients in the series solution of the differential equation and obtain a_2 5mks

c. Solve the initial value problem $y' + y = 1,\ y(0) = 1$ by La[lace transforms 3mks

QUESTION FOUR

a. Solve the system below by Laplace transforms 5mks

$$y'' + z + y = 0$$
$$z' + y' = 0$$

Given

$$y(0) = 0,\quad y'(0) = 0,\quad z(0) = 1$$

b. Obtain a_0 *and* a_n *and* b_n for the Fourier series represented by $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$ 8mks

c. Applying Laplace transform, find the solution of the differential equation $y'' + y = \sin t$, satisfying the initial condition $y(0) = 1,\ y'(0) = 0$ 7mks

QUESTION FIVE

a. Obtain the Fourier series expansion of the rectified sine wave $f(t) = |\sin t|$ 5mks

b. Evaluate the laplace transform of $te^{-t}\sin 2t$ 5mks

c. Identify the nature of the singular points of the equation

$$3x(x-2)^2y'' + 6(x-2)y' + 3(x+3)y = 0$$ 6mks

d. Find the Laplace transform of $\frac{\sin 2t}{t}$ 4mks