CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

MATH 326: METHODS OF APPLIED MATHS 1

STREAMS: **TIME:2 HOURS**

DAY/DATE: THURSDAY 13/04/2023 **11.30 A.M. –2.30 P.M.**

INSTRUCTIONS

Answer question one and any other two questions
Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

a. Identify the nature of the singular points of the equation $4x^2y'' + 5xy' + \frac{1}{2}(x-1)y = 0$ (5 marks)

b. Find the series solutions about $x = 0$ of $y'' + y = 0$ (6 marks)

c. Decompose $\frac{s+3}{(s-2)(s-3)}$ into partial fractions hence evaluate $L^{-1}\left(\frac{s+3}{(s-2)(s-3)}\right)$ (5 marks)

d. Find the sine Fourier series for the function $f(x) = 1,\ in\ 0 < x < \pi$ (5 marks)

e. Determine the nature of the singular points of the differential equation $\frac{3}{2}xy'' + y' + \frac{1}{2}y = 0$, Hence Find the roots of it's indicial equation (6 marks)

f. Evaluate the $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$ (3 marks)

QUESTION TWO

a. The Legendre`s equation has the form $(1-z^2)y'' - 2zy' + l(l+1)y = 0$, where l is a constant and z is the dependent variable,

i. Show that , $z=0$ is a an ordinary point and $z=\pm 1$ is a regular singular point of the equation (5 marks)

ii. Show that the Legendre`s equation has a regular singularity as $|z|=\infty$ (7 marks)

b. A periodic function f(t) of period 2π is defined by $f(t)=t^2+t,\ -\pi<t<\pi$. Evaluate b_n, a_0 *and* a_n and obtain the Fourier series expansion of the function (8 marks)

QUESTION THREE

a. Using the Laplace transforms, to evaluate $\int_0^\infty te^{-3t}\sin t\,dt$ (5 marks)

b. Given the Bessel`s differential equation $x^2y''+xy'+\left(x^2-n^2\right)y=0$, about the point x = 0.

i. Obtain the roots of the indicial equation of the differential equation (7 marks)

ii. Find the recurrence relation satisfied by coefficients in the series solution of the differential equation and obtain a_2 (5 marks)

c. Solve the initial value problem $y'+y=1,\ y(0)=1$ by La[lace transforms (3 marks)

QUESTION FOUR

a. Solve the system below by Laplace transforms (5 marks)

$$y''+z+y=0$$
$$z'+y'=0$$

Given

$$y(0)=0,\quad y'(0)=0,\quad z(0)=1$$

b. Obtain a_0 *and* a_n *and* b_n for the Fourier series represented by $f(x)=\begin{cases}2, & -2<x<0\\ x, & 0<x<2\end{cases}$ (8 marks)

c. Applying Laplace transform, find the solution of the differential equation $y''+y=\sin t$, satisfying the initial condition $y(0)=1,\ y'(0)=0$ (7 marks)

QUESTION FIVE

a. Obtain the Fourier series expansion of the rectified sine wave $f(t)=|\sin t|$ (5 marks)

b. Evaluate the laplace transform of $te^{-t}\sin 2t$ (5 marks)

c. Identify the nature of the singular points of the equation

$$3x(x-2)^2y''+6(x-2)y'+3(x+3)y=0$$ (6 marks)

d. Find the Laplace transform of $\frac{\sin 2t}{t}$ (4 marks)