CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 346: STATISTICAL INFERENCES I

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: THURSDAY 31/08/2023 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS:

- Answer all the Questions.

<u>Question One</u> **(30 marks)**

a) Differentiate between an estimator and an estimate (3marks)

b) A machine produces steel rods with lengths that are normally distributed with mean μ and variance σ^2.

A quality control inspector uses a gauge to measure the length, x centimeters, of each rod in a random sample of 100 rods from the machine's production. The summarized data are as follows.

Σ x =1040.0 Σ x^2=11,102.11

i) Calculate unbiased estimates of μ and σ^2 (3 marks)

ii) Construct a 99% confidence interval for μ. (4 marks)

C) Briefly explain the below properties of Maximum Likelihood estimators (MLEs)

i) Consistency (2 marks)

ii) Invariance (2 marks)

d) A random sample from a *Bin (n,p)* distribution yields the following values:
4, 2, 7, 4, 1, 4, 5, 4
Find method of moments estimates of *n* and *p* using *x* and s^2 (4 marks)

e) A company wishes to estimate the mean claim amount for claims under a certain class of policy during the past year. Extensive past records from previous years suggest that the standard deviation of claim amounts is likely to be about Kshs 45. If the company wishes to estimate the mean claim amount such that a 95% confidence interval is of width " ± Kshs.5 ", determine the sample size needed to achieve this accuracy of estimation. (6 marks)

f) A random variable X is believed to have probability density function, $f(x)$, where:

$$f(x) = 3\lambda^3(\lambda+x)^{-4} \quad x > 0$$

In order to test the null hypothesis $\lambda = 50$ against the alternative hypothesis $\lambda = 60$, a single value is observed. If this value is greater than 93.5, H_0 is rejected.

i) Calculate the size of the test. (3 marks)
ii) Calculate the power of the test (3marks)

<u>Question Two</u> (20 marks)

a) Outline the 4 major Steps of applying maximum likelihood (6 marks)

b) The ordered remission times (in weeks) of 20 leukemia patients are given in the table:

1	1	2	2	3
4	4	5	5	8
8	8	11	11	12
12	15	17	22	23

Suppose the remission times can be regarded as a random sample from an exponential distribution with density:

$$f(x; \lambda) = \lambda e^{-\lambda x} \quad x>0$$

Determine the maximum likelihood estimator λ(hat) of λ (8 marks)

c) Calculate the spearman's rank correlation coefficient for the below data (6 marks)

x	10	8	12	15	8	10
y	7	4	6	7	9	8

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<u>Question Three</u>(20 marks)

a) Carry out a statistical test to assess whether the standard deviation of the heights of 10-year-old children is equal to 3cm, based on the random sample of 5 heights in cm given below. Assume that heights are normally distributed (5 marks)

124, 122, 130, 125, 132

b) A coin is selected at random from a pair of coins and tossed. Coin 1 is a double-headed coin (*ie*a head on both sides). Coin 2 is a standard unbiased coin. The result of the toss is a head. What is the probability that it was coin 1 which was tossed? Hint: use bayes theorem

(5 marks)

c) In a small survey, a random sample of 50 people from a large population is selected. Each person is asked a question to which the answer is either \Yes" or \No." Let the proportion in the population who would answer \Yes" be θ: Our prior distribution for θ is a beta $(1:5; 1:5)$ distribution. In the survey, 37 people answer \Yes."

i) Find the prior mean and prior standard deviation of θ (3 marks)

ii) Find the likelihood (2 marks)

iii) Find the posterior distribution of θ (2 marks)

iv) Find the posterior mean and posterior standard deviation of θ (3 marks)
