# MATH 347: PROBABILITY MODELLING

QUESTION ONE

a) Define the following processes

i)Compound Poisson process (2 marks) ii)Bernoulli process (2 marks)
iii) Renewal process (2 marks)

b) State four scenarios that a Bernoulli process can be used to model. (4 marks)

c) The counter of a bank branch performs transactions with a mean time of 2 minutes. The customers arrive at a mean rate of 20 customers/hour. If we assume that arrivals follow an exponential process and that the service time is exponential, determine:

i. Percentage of the time the bank teller is idle (3 marks)
ii. Mean waiting time of the customers in the queue. (4 marks)
iii. Number of customers in the system. (3 marks)

d)Explain what each of the notations A/B/c/K denote in the Kendall-Lee standard notation system used to describe and classify a queueing node. (4 marks)

e) The time between arrivals at Ndagani restaurant is exponential with mean 5 minutes. The restaurant opens at 11:00 A.M. Determine the probability of having 10 arrivals in the restaurant by 11:12 A.M, given that 8 customers arrived by 11:05 A.M. (6 marks)

QUESTION TWO

a) Consider the Markov chain having states 0, 1, 2, 3, 4 and

$$\mathbf{P} = \begin{Vmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{Vmatrix}$$

Explain which states are recurrent and which are transient. (4 marks)

b)Let N(t) be a Poisson process with intensity $\lambda$ =2, and let $X_1$, $X_{2,\ldots}$ be the corresponding interarrival times.

i)Find the probability that the first arrival occurs after t=0.5,i.e $P(X_1 > 0.5)$.(3 marks) ii)Given that we have no arrivals before t=1,find $P(X_1 > 3)$.(3 marks)

c) A model of social mobility of families posits three different social classes (strata), namely “lower”, “middle”, and “upper”. The transitions between these classes (states) for a given family are governed by the following transition matrix:

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 2/3 \end{pmatrix}$$

Determine the steady state stationary distribution of the below problem. (10 marks)

QUESTION THREE

a) State the general assumptions of birth-and-death processes.(6 marks)

b) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.

we can transform this model into a Markov chain by saying that the state at any time is determined by the weather conditions during both that day and the previous day. In other words, we can say that the process is in state 0 if it rained both today and yesterday,

state 1 if it rained today but not yesterday, state

2 if it rained yesterday but not today, state 3 if it

did not rain either yesterday or today.

The preceding would then represent a four-state Markov chain having a transition probability matrix.

$$\mathbf{P} = \begin{Vmatrix} 0.7 & 0 & 0.3 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.4 & 0 & 0.6 \\ 0 & 0.2 & 0 & 0.8 \end{Vmatrix}$$

Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday?(4 marks)

c)The mood of an individual is considered as a three-state Markov chain having a transition probability matrix.

$$\mathbf{P} = \begin{Vmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{Vmatrix}$$

In the long run, what proportion of time is the process in each of the three states?(10 marks)

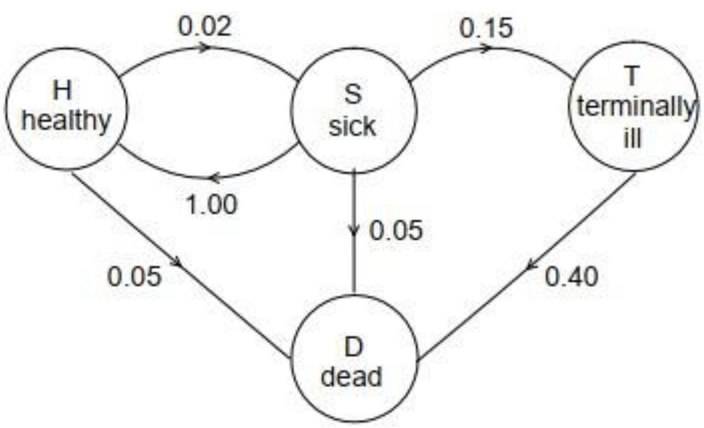
QUESTION FOUR

a)What is first-order interarrival time in a Bernoulli process. (2 marks)

b)Assume that a park attendant takes an average of 15 sec to distribute brochures but the distribution time varies depending on whether park patrons have questions relating to park operating policies. Given average arrival rate of 180 vehicles per hour. Compute

i. Average length of queue (2 marks)
ii. Average waiting time in the queue. (2 marks)
iii. Average time spent in the system (2 marks)

c) Consider the following Health, Sickness, Death model with the addition of an extra "Terminally ill" state, T. The rates given are per year.




**i)** Calculate the expected holding time in state S (2 marks)
**ii)** Calculate the probability that a sick life goes into state D when he leaves the sick state.(2 marks)
**iii)** Calculate the expected future lifetime of a healthy life.(8 marks)

QUESTION FIVE

a) Explain four key properties of the Bernoulli process,(include the formulae). (8 marks)

b) Consider a Markov process with state space S =□0,1, 2□ and transition matrix, P:

$$P = \begin{pmatrix} p & q & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ p-\frac{1}{2} & \frac{7}{10} & \frac{1}{5} \end{pmatrix}$$

i)What can you say about the values of p and q ?(3 marks) ii)Calculate the transition probabilities $p_{i\,j}^{(3)}$(5 marks)

iii)Draw the transition graph for the process represented by P. (4 marks)