CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION

MATH 411: DIFFERENTIATE GEOMETRY

STREAMS: B.ED **TIME: 2 HOURS**

DAY/DATE: FRIDAY 01/09/2023 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS

Answer Question**ONE** and any other **TWO** Questions

QUESTION ONE (30 MARKS)

a) Find the volume of the parallelepiped formed by the vectors $\vec{a} = (2,1,1), \vec{b} = (1,-1,2)$ and $\vec{c} = (0,-2,3)$ (4 marks)

b) Determine the cartesian equation of the curve $\vec{r}(t) = (cos^2t, sin^2t)$ (4 marks)

c) Show that a particle whose motion is given as$\vec{r} = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{t}{\sqrt{2}}\right)$ has a unit speed (4 marks)

d) Calculate the length of the curve *x=2t*, *y=4sin3t* and *z=4cos3t*; $0 \leq t \leq 2\pi$ (5 marks)

e) Evaluate $\int_0^2 (6t^2 i - 4tj + 3k)dt$ (3 marks)

f) Show that if $\vec{u}(t) = g(t)\hat{i} + h(t)\hat{j}$ then $\frac{d[f(t)\vec{u}(t)]}{dt} = f'(t)\vec{u}(t) + f(t)\vec{u}'(t)$(5marks)

g) Given that $\vec{r}(t) = t^2\hat{i} + \sqrt{5-t^2}\,\hat{j}$ where t is time in seconds, find the velocity, acceleration and speed (5marks)

QUESTION TWO (20 MARKS)

a) Find the equation of the osculating plane to the given curve $\vec{r} = (t^3, t^2, t)$ at t=2 (8 marks)

b) The parametric equation of a curve is x=3cos2t, y=3sin2t, z=6t. find the length of the arc from $0\ to\ \pi$ (6 marks)

c) Prove the Frenet-Serret formula for a space curve $\vec{r} = f(t)$

i) $\frac{d\bar{T}}{ds} = k\bar{N}$ (3 marks)

ii) $\frac{d\bar{B}}{ds} = \bar{\iota}\bar{N}$ where $\bar{T}, \bar{N}, \bar{B}, \bar{k}$ and $\bar{\iota}$ have the usual meaning (3 marks)

QUESTION THREE (20 MARKS)

a) Find the tangent and normal line passing through P(x,y) on the curve $\vec{r}(t) = (2sint - sin2t, 2cost - cos2t)$ at the point corresponding to $t = \frac{\pi}{4}$ (11 marks)

b) If $\vec{r} = (\frac{4}{5}cost, 1 - sint, \frac{-3}{2}cost$ find the curvature k (9 marks)

QUESTION FOUR (20 MARKS)

a) Find the arc length of the spiral $\vec{r}(t) = (e^{kt}cost, e^{kt}sint)$ (7 marks)

b) A parallelogram is determined by the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Given that $\overrightarrow{PQ} = (4,3,-2)$ and $\overrightarrow{PR} = (5,5,1)$, find;

i) The area of the parallelogram (3 marks)

ii) The angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ (3 marks)

iii) Show that $\vec{r}(s) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{3}{\sqrt{3}}\right)$ is a unit speed curve and find its SerretFrenet apparatus (7 marks)

QUESTION FIVE (20 MARKS)

A space curve C is given by $\bar{r} = 5sint\hat{\imath} + 5cost\hat{\jmath} + 12t\hat{k}$, calculate the following at a point P on the curve where $t = \frac{\pi}{2}$

a) Unit tangent (T) (4 marks)

b) The curvature (k) and radius of curvature (ϱ) (4 marks)

c) The unit normal ($\bar{N}$) (4 marks)

d) The Binomial ($\bar{B}$) (4 marks)

e) The torsion τ and the radius of the torsion δ (4 marks)
