**CHUKA UNIVERSITY**

# UNIVERSITY EXAMINATIONS

**THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN PHYSICS, BACHELOR OF EDUCATION SCIENCE**

**PHYS 317: MATHEMATICAL PHYSICS 1**

**STREAMS: BSC (PHYS), BED (SCI) Y3S1** **TIME: 2 HOURS**

**DAY/DATE: MONDAY 10/12/2018** **2.30 PM – 4.30 PM**

**INSTRUCTIONS:**

- **Answer question ONE and any other TWO questions**
- **Use of mathematical tables and unprogrammable calculator is permissible.**

**Question One;**

a. Differentiate between
   i. A linear and a non-linear ordinary differential equation
   ii. A homogeneous and an inhomogeneous ODE. (4 marks)

b. State and write the mathematical formulae for the following functions
   i. Gamma
   ii. Beta
   iii. Bessel (6 marks)

c. Differentiate between an eigenvalue and an eigenvector (2 marks)

d. State the Cauchy's theorem and hence write the Cauchy integral giving two of its applications. (4 marks)

e. State what is meant by two vectors being orthogonal giving the mathematical expression for it. (2 marks)

f. State the Green, Stoke and divergence theorems giving the mathematical expression for each of them. (6 marks)

g. Write the Laplace equation in rectangular and spherical coordinates. (3 marks)
h. What is meant by the gradient, divergence and curl of a vector? (3 marks)

**Question Two;**

a. The force $F$ in Newtons acting on a body at a distance $x$ metres from a fixed point is given by: $F = 2x + 3x^2$. If work done $w = \int_{x_1}^{x_2} F dx$ determine the work done when the body moves from the position when $x = 1$m to that when $x = 4$m. (4 marks)

b. If the distance moved by a body is given by $x = 3\tan\theta$, the angular velocity, ω, is $d\theta\theta/dt$ and the velocity v is $dx/dt$, show that $\omega = \frac{v}{3}\cos^2\theta$ (4 marks)

c. A point on a curve is given by $x = 7\,cost + 3.5\cos 2t$, $y = 7\,sint - 3.5\sin 2t$. Express $\frac{d^2y}{dx^2}$ in terms of t. (4 marks)

d. If $\varnothing = f(r,\theta)$ and $\varnothing = (Ar^n + br^{-n})\sin(n\theta + \alpha)$ where A, B n and α are constants, show that

$$\frac{\partial^2\varnothing}{\partial r^2} + \frac{1}{r}\frac{\partial\varnothing}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varnothing}{\partial\theta^2} = 0$$ (8 marks)

**Question Three;**

a. The following equation represents the undamped simple harmonic motion. Obtain the general solution $\frac{d^2y}{dx^2} + 4y = 0$ (6 marks)

b. The following equation represents the damped simple harmonic motion. Obtain the general solution $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2 = 0$ (7 marks)

c. Find the general solution for the following differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x^2$$ (7 marks)

**Question Four;**

a. The elastic energy of a volume $V$ of material is $q^2V/(2EI)$, where $q$ is its stress and $E$ and $I$ are constants. Find the elastic energy of a cylindrical volume of radius $r$ and length $l$ in which the stress varies directly as the distance from its axis, being zero at the axis and $q0$ at the outer surface. (12 marks)

b. A right circular cone of height $h$ and base radius $a$ is cut into two pieces along a plane parallel to and distance $c$ from the axis of the cone. Find the volume of the smaller piece. (8 marks)

**Question Five;**

a. A capacitor C is charged by applying a steady voltage E through a resistance R. The p.d. between the plates, V, is given by the differential equation $CR\frac{dV}{dt}+V=E$. Solve the equation for V given that V=0 when t=0 and evaluate V when E=20 volts, C=25μF, R=300KΩ and t=2s. (6 marks)

b. The charge q on a capacitor in a certain electrical circuit satisfies the differential equation $\frac{d^2q}{dt^2}+3\frac{dq}{dt}+4q=0$. Initially, (when t=0), q=Q and $\frac{dQ}{dt}=0$ Find an expression for the charge, $q$, in the circuit. (6 marks)

c. The instantaneous current, $I$, passing through a solution, in a circuit of resistance R and inductance L, whose dielectric constant is to be measured is given by $\frac{di}{dt}+\frac{R}{L}i=\frac{V_0}{L}\,sinpt$, whret is time and $V_0$ and p are constants. Solve the equation for $i$. (8 marks)

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