CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF SCIENCE

PHYS 362: THERMAL AND STATISTICAL PHYSICS

Streams: BED (SCI) & BSC **TIME: 2 HOURS**

DAY/DATE: MONDAY 17/12/2018 **2.30 P.M. – 4. 30 P.M.**

INSTRUCTIONS:

- Answer Question One in Section A and any other Two Questions in Section B
- Do not write anything on the question paper
- This is a closed book exam, No reference materials are allowed in the examination room
- There will be No use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

Data;

$$ds \equiv \frac{dQ}{T}$$

Stirling's approximation $\ln N! \approx N \ln N - N$

Energy of an ideal gas $U = \frac{3}{2}NkT$

$$\int_0^\infty e^{-(ax^2)}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty x^2 e^{-ax^2}dx = \frac{\sqrt{\pi}}{4}$$

$$\int_0^\infty x^3 e^{-(ax^2)}\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^4 e^{-(ax^2)}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

QUESTION ONE

(a) Define a state variable in thermodynamics. **(3 marks)**

(b) Consider a system consisting of N distinguishable particles, each of which has only three possible energy states. The energies of the states are, $E_0 = 0, E_1 = \varepsilon, E_2 = 2\varepsilon$. The system is in equilibrium with a heat reservoir which is at temperature T.

i. What is the partition function Z for a single particle? (Note that it has only three energy levels) (2 marks)

ii. What is the partition function for the entire system of N particles? (Note that the N particles are distinguishable). You can give your answer in terms of Z from part (i) above.(2 marks)

(c) State the postulates upon which statistical mechanics is based.(4 marks)

(d) Boltzmann's hypothesis defines entropy S in terms of the number of microstates as $S = k_B \ln \Omega$. Show that the entropy for a combined system of two subsystems 1 and 2 is $S = S_1 + S_2$, where S_1 and S_2 is the entropy of 1 and 2 respectively. (4 marks)

(e) What are fermions and bosons? Distinguish between them and give two examples of each. (6 marks)

(f)

i. What is an ensemble? (1 mark)

ii. Distinguish between the different types of ensembles in statistical mechanics (6 marks)

QUESTION TWO (20 marks)

a. The Gibbs free energy is defined as $G = E - TS + pV$, where E is internal energy, T is temperature, S is entropy, p is pressure and V is volume.

i. Show that $dG = -SdT + Vdp + \mu dN$(3 marks)

ii. Use the equation given in (i) to show that $V = \left.\frac{\partial G}{\partial p}\right)_{T,N}$(3 marks)

iii. Use the equation given in (i) to derive the Maxwell's relation $-\left.\frac{\partial S}{\partial p}\right)_{T,N} = \left.\frac{\partial V}{\partial T}\right)_{p,N}$ (4 marks)

b. Taking S and V to be Independent variables with x=T and y=V, derive the Maxwell's thermodynamic relation $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ stating from the relation $dU = Tds - pdV$ for an infinitesimal reversible process. (10 marks)

QUESTION THREE (20 marks)

Consider a particle in one-dimensional box of width, L, and infinite barriers. For such a system the energy eigen values are,

$$E_n = \frac{\hbar^2\pi^2}{2mL^2}n^2, \quad n = 1,2,3 \ldots.$$

i. Show that, the partition function can be written as, $Z = \frac{L}{\lambda_D}$ where $\lambda_D = \sqrt{\frac{2\pi\hbar^2}{mkT}}$ is the de Broglie wavelength. (9 marks)

ii. Calculate the Helmholtz free energy (3 marks)

iii. Calculate the entropy. (4 marks)

iv. Derive the equation of state by extending the discussion above to a 3-D box.

(4 marks)

QUESTION FOUR (20 Marks)

a. What do you understand about the principle of equipartition theorem? (2 marks)

b. Consider a huge number of systems, N, in thermal contact and let N_1 be in microstate 1, N_2 in microstate 2... and N_i be in microstate i, show that the entropy per system is, $S = -k\sum_i P_i \ln P_i$(5 marks)

c. Suppose system A is in contact with a with a huge reservoir with which the system can exchange both heat and particles. Show that the probability that A is in the ith state is,

$$= \frac{e^{-(E_i - \mu N_i)/k_B T}}{\Xi}$$

Where $\Xi = \sum_j e^{-(E_i - \mu N_i)/k_B T}$ is the grand partition function.(13 marks)

QUESTION FIVE (20 marks)

Consider a two dimensional monoatomic gas i.e. gas molecules can move freely on a plane but are confined within an area A. Assume that the molecules obey Boltzmann statistics and that molecules are point particles which exert no force on one another when they collide.

i. Show that the partition function for a two dimensional monatomic gas of N particles is given by $Z = \frac{2mA\pi\, k_B T}{h^2}$(5 marks)

ii. Find the velocity distribution function (4 ½ marks)

iii. Use the result from (ii) above to calculate the force per unit length which the gas exerts on its 2-D container. Express this pressure in terms of the temperature to get the equation of state. (4 ½ marks)

iv. Using the partition function in part I above, derive the expression for the entropy and heat capacity of this two dimensional monatomic gas. (6 marks)
