CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OFBACHELOR OF SCIENCE AND BACHELOR OF EDUCATION SCIENCE

PHYS 415: QUANTUM MECHANICS II

STREAMS: BSC, BED (SCIENCE) **TIME: 2 HOURS**

DAY/DATE: THURSDAY 13/12/2018 **11.30 A.M -1.30 P.M**

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Clearly show your working.
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The following constants may be useful.

Gravitational constant G = 6.672 x 10^{-11}N m^2 kg^{-2}

Permittivity of free space $\varepsilon_0 = 8.85\ x\ 10^{-12}$

Boltzman constant $k_B = 1.38x10^{-23}J/K$=8.63 x 10-5 eV/K)

Planks constant $h = 6.63x10^{-34}$J.S

Mass of an electron = 9.1 x 10^{-31} kg

Mass of a proton = 1.67 x 10^{-27} kg

Electronic charge = 1.6021 x 10^{-19}C

Reduced Plancks constant $\hbar = 1.05x10^{-34}J.S.$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$$

$$\int Sin^2\,\theta d\theta = \frac{\theta}{2} - \frac{Sin\ 2\theta}{4}$$

QUESTION ONE

(a) Giving one example in each case, state how particles and observables are represented in Quantum Mechanics. (2 marks)

(b) A vector A, in Cartesian space takes the form $\vec{A} = iA_x + jA_y + kA_z$ write this vector in Ket notation. (2 marks)

(c) (i) Determine whether the matrix below is suitable in Quantum Mechanics. (3 marks)

$$\begin{bmatrix} 0 & \frac{-i\hbar}{2} \\ \frac{i\hbar}{2} & 0 \end{bmatrix}$$

(ii) Explain your answer in b (i) above. (2 marks)

(d) Briefly explain what is involved in the perturbation theory. (3 marks)

(e) Describe the Stark effect. (2 Marks)

(f) Calculate the probability of finding a hydrogen atom electron within the radius of a_0 given that its ground state wave function is;

$\varphi_o(r) = \frac{1}{\sqrt{\pi {a_0}^3}} e^{\frac{-r}{a_0}}$ (6 marks)

(g) Calculate the relative energy shift between the unperturbed Is state and the corrected Is state due to the gravitational potential between the proton and the electron in the hydrogen atom. (6 marks)

(h) Describe the Zeeman effect. (2 marks)

(i) State Pauli's exclusion principle (2 mark)

QUESTION TWO

(a) Distinguish between degenerate and non-degenerate states. Give one example in each case. (4 marks)

(b) Show that the amplitude $\boldsymbol{a_m}$ of the 1st order perturbation for the non-degenerate case can be given as; $a_m = \frac{\langle \Psi m | H' | \omega_0 \rangle}{E_0 - E_m}$ (8 marks)

(c) The perturbation for a harmonic oscillator is $H' = ae^{-\beta x^2}$. Determine the energy correction for the ground state energy. Take $|\Psi_0\rangle = (\alpha/\sqrt{\pi})^{\frac{1}{2}} e^{-\alpha^2 \frac{x^2}{2}}$. (8 marks)

QUESTION THREE

(a) Derive the expression for the first energy correction resulting from first order perturbation. (8 marks)

(b) Calculate the corrected second eigenvalue to first order perturbation in the finite well problem. Take $\Psi_n = \sqrt{\frac{2}{L}} Sin\ \frac{2\pi x}{L}$ (12 marks)

QUESTION FOUR

(a) (i) Distinguish between the approach taken by the Perturbation theory, variation method and WKB methods in approximating solutions to the Schrodinger equation. (4marks)

(ii) Using variation theory show that the eigenvalue with an unknown wave function $\bar{\psi}$ gives the upper bound to the energy of the known wave function ψ. (5 marks)

(b) The Hamiltonian of a quantum oscillator can be given as $H = -\frac{\hbar^2}{2m}\nabla + \frac{1}{2}m\omega^2 x^2$, in one dimension. Calculate the lowest energy. (11 marks)

QUESTION FIVE

(a) Work out the following commutation relations:-

(i) $[L_x, L_y]$ (2 marks)

(ii) $[L_x, L_z]$ (2 marks)

(iii) $[L_y, L_z]$ (2 marks)

(b) The states in a hydrogen atom has the principal quantum number n=4. Calculate the:-

(i) Corresponding angular momentum quantum number (2 marks)

(ii) Angular momentum **L (**2 marks**)**

(iii) Component of **L** in the z direction (2 marks)

(iv) Angle between **L** and z direction (2 marks)

(c) The Helium atom has more than one electron. Set up its Hamiltonian,taking this fact into account and clearly explaining each term. (6 marks)
