CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE (PURE MATHEMATICS)

MATH 803: GENERAL TOPOLOGY I

STREAMS: MSC (PURE MATHS) **TIME: 3 HOURS**

DAY/DATE: THURSDAY 05/12/2019 **11.30 A.M. – 2.30 P.M.**

INSTRUCTIONS:

- **Answer any three questions**
- **Sketch maps and diagrams may be used whenever they help to illustrate your answer**
- **Do not write on the question paper**
- **This is a closed book exam, no reference materials are allowed in the examination room**
- **There will be No use of mobile phones or any other unauthorized materials**
- **Write your answer legibly and use your time wisely**

QUESTION ONE (20 MARKS)

(a) Let A be a compact subset of a Hausdorff space X and suppose $p \in X/A$, show that there exists open sets G and H such that $p \in G, A \subset H = \emptyset$ [4 marks]

(b) Show that an open interval $A = (0, 1)$ on the real line $\mathbb{R}$ with the usual topology is not subsequentially compact. [3 marks]

(c) Prove that every bounded closed interval $B = [a, b]$ is countably compact. [3 marks]

(d) Let $G \cup H$ be a disconnection of B. Show that $B \cap G$ and $B \cap H$ are separated sets [5 marks]

(e) Let A be a subset of a Topological space X. prove that if X is sequentially compact then X is countably compact. [5 marks]

QUESTION TWO (20 MARKS)

(a) Prove that every arcwise connected set A is connected [4 marks]

(b) Explain what is meant by a subset of a topological space X being compact and hence show that every compact subset A of Hausdorff space is closed. [4 marks]

(c) Show that a continuous image of a compact set is also compact. [4 marks]

(d) Explain what is meant by a class $\{A_i\}$ of sets having a finite intersection property and hence show if or not the class $A = \left\{(0,1), \left(0, \frac{1}{2}\right), \left(0, \frac{1}{3}\right), \ldots\right\}$ of intervals in $\mathbb{R}$ has a finite intersection property [4 marks]

(e) Let $\mathcal{G}$ be a base for a second countable space X. Prove that $\mathcal{G}$ is reducible to a countable base for X. [4 marks]

QUESTION THREE (20 MARKS)

(a) Define a continuous map and show that a map is continuous if and only if the inverse image of an open set is open [5 marks]

(b) Show that a topological space is a Hausdorff space if and only if any net has at most one limit. [5 marks]

(c) State without proof Tiestze Extension Theorem [2 marks]

(d) Prove that a subset of a countable set is countable [4 marks]

(e) Prove that the property of regular space is hereditary [4 marks]

QUESTION FOUR (20 MARKS]

(a) Consider the topology $\tau = \{X, \emptyset, \ \{a\}, \{b, c\}\}$ $on\ X = \{a, b, c\}$ and the topology $\tau^* = \{Y, \emptyset, \{0\}\}$ on $Y = \{u, v\}$. Determine the defining subbase of product topology $X_x Y$. [4 marks]

(b) Prove that every projection mapping on a product space X is a bicontinous mapping [4 marks]

(c) Prove that a completely regular is also regular [3 marks]

(d) State and proof the Urysohn's Lemma. [9 marks]
