

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 821: ORDINARY DIFFERENTIAL EQUATIONS I

STREAMS: M.Sc (APPLIED MATHS)

TIME: 3 HOURS

DAY/DATE: THURSDAY 8/08/2019

8.30 A.M - 11.30 A.M.

INSTRUCTIONS

- Answer any **THREE** Questions
- You may use advanced calculators.
- Do not write anything on the question paper

QUESTION ONE - (20 MARKS)

- (a) Write brief notes on linear differential equations. [4 Marks]
- (b) Use variation of parameters to solve the initial value problem given that $y = x$ and $y = e^x$ are solutions to the homogeneous function $(x - 1)y'' - xy' + y = (x - 1)^2$; $y(0) = 3, y'(0) = -6$. [8 Marks]
- (c) Solve the homogeneous differential equation using the method of immerse operator. $(2D^2 + D - 1)y = 16 \cos 2x$ [8 Marks]

QUESTION TWO - (20 MARKS)

- (a) (i) State the necessary and sufficient condition that n functions are a fundamental set. [2 Marks]
- (ii) What is the significance of the condition i n a (i) above. [2 Marks]
- (b) Find the general solution of the differential equation given that $y = e^x$ is a solution of the homogeneous equation $xy'' - (2x + 1)y' + (x + 1)y = x^2$. [10 Marks]
- (c) Solve the following differential equations using the method indicated.
- (i) $\dot{x} + x = \epsilon x^2, x(0) = 1$ Regular perturbation method. [3 Marks]
- (ii) $\dot{x} + x = 1; x(0) = 0$ singular perturbation method [3 Marks]

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QUESTION THREE [20 MARKS]

(a) Use the reduction of order method to solve the differential equation given that $y = x$.
 $x^2y'' - 4xy' + 4y = 0$ [10 Marks]

(b) Using the method of undetermined coefficients, solve the system of differential equations. [10 Marks]

$$\begin{aligned} \dot{x}_1 &= 3x_1 - 3x_2 + 2 \\ \dot{x}_2 &= -6x_1 - t \end{aligned}$$

QUESTION FOUR: [20 MARKS]

(a) State and prove the principle of superposition of linear differential equations. [4 Marks]

(b) Solve the system [8 Marks]

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

(c) Find the fundamental set of the differential equation given that $y = x$ is a solution.
 $X^2y'' - 3xy' + 3y = 0$ [8 Marks]

QUESTION FIVE: [20 MARKS]

(a) (i) Define a solution of linear differential equations. [2 Marks]

(ii) Explain the difference between the general solution and complete solution of a linear differential equation. [2 Marks]

(b) (i) Show that $\{e^{-3t}, e^{-t}, e^{-6t}\}$ is a fundamental set. [2 Marks]

(ii) Find the differential equations associated with the set in b(i) above. [4 Marks]

(c) Solve the third order Cauchy-Euler equation $2x^2y + 2xy' - 3y = 0$ for $x > 0$. [10 Marks]

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