CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 824: PARTIAL DIFFERENTIAL EQUATIONS II

STREAMS: MSC MATHS **TIME: 3 HOURS**

DAY/DATE: FRIDAY 06/12/2019 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS:

- **Answer any FOUR questions.**
- **Adhere to instructions on the answer booklet.**

QUESTION ONE

(a) Prove that

(i) $\sqrt{n+1}$ = n (3 marks)

(ii) $\sqrt{\frac{1}{2}} = \sqrt{\pi}$ (3 marks)

(b) Evaluate $\int_0^\infty \sqrt{x}\ e^{-\sqrt[3]{x}}\ dx$ by the gamma function. (3 marks)

(c) Show that $\int_0^{\frac{\pi}{2}} \tan\theta d\theta = \frac{\pi}{2} \sec\left(\frac{p\pi}{2}\right)$ and indicate the restrictions on the values of p. (3 marks)

(d) Determine the mass of an octant of the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2^2}{c^2}$=1, the density at any point being $P = kxy$Z. (3 marks)

QUESTION TWO

(a) Obtain the Fourier transform of

$$\int(x) = \begin{cases} 1 - x^2, |x| \leq 1 \\ 0, \quad |x| > 1 \end{cases}$$ (3 marks)

(b) Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$ (3 marks)

(c) Evaluate the Fourier cosine and sin c transform of

$f(x) = x^{n-1}$ (5 marks)

(d) Show that $\int_0^\infty \frac{x^2 dx}{(x^2+1)^2} = \frac{\pi}{4}$ by Parseval's identity. (4 marks)

QUESTION THREE

(a) Solve the heat equation

$U_t = U_{xx}$ by the Fourier sine transforms subject to the conditions

(i) $u = 0$ when $x = 0, t > 0$

(ii) $u = \begin{cases} 1, 0 < x < 1 \\ 0, \quad x \geq 1 \end{cases} \Big| \, when \; t = 0$

(iii) $u(x,t)$ is unbounded (6 marks)

(b) Use Fourier transform to solve $u_{tt} = \propto^2 u_{xx}, -\infty < x < \infty, t \geq 0$
With conditions

$u(x,0) = f(x), u_t(x,0) = g(x),$

$u_x \to 0 \; as \; x \to \pm\infty.$ (6 marks)

(c) Find the finite Fourier sine and cosine transforms of

$f(x) = 1 \;\; in \; (o, \pi)$ (3 marks)

QUESTION FOUR

(a) Solve the Pde

$u_t = ku_{xx}$ for $0 \le x < \infty, t > 0$ given the conditions

$u(x, 0) = 0,$ $for\ x \ge 0$

$u_x(0, t) = -a$ and $u(x, t)$ is bounded. (5 marks)

(b) Solve the Pde

$u_t = u_{xx}$ given

$u(0, t) = 2x$ $where$ $0 < x < 4, t > 0$ using finite Fourier transforms. (5 marks)

(c) Find the finite Fourier since transform of $f(x) = 1$ in $(0, \pi)$, Hence prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$ (5 marks)

QUESTION FIVE

Solve the following partial differential equations by the D-operator

(i) $u_{xx} - u_{yy} + u_x + 3u_y - 2u = x^2 y.$ (8 marks)

(ii) $u_{xx} + 9u_{yy} - 6u_{xy} - 4u_x + 12u_y + 4u = 2e^{2x}\sin(y + 3x).$ (7 marks)
