CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 825: NUMERICAL ANALYSIS I

STREAMS: MSC (APPLIED MATHEMATICS) **TIME: 3 HOURS**

DAY/DATE: FRIDAY 09/08/2019 **2.30 PM – 5.30 PM**

INSTRUCTIONS:

- **Answer any Three Questions**
- **You may use advanced calculators**

QUESTION ONE (20 MARKS)

(a) Use the method of bilinear approximation to approximate $f(1,\ 0.25)$ given that $f(0.0) = 1, f(0,1) = 2, f(1,0) = 3\ and\ f(1,1) = 5$ [4 marks]

(b) (i) State two assumptions made in using the Gauss Jacob method to solve systems of linear equations. [2 marks]

(ii) Write the computational difference between Gauss Jacob and Gauss Siedel methods and state the significance of the difference. [3 marks]

(iii) Solve the system of linear equations using the Gauss Jacob's method with 5 iteration. Give your answer to 2 s.f [11 marks]

$x_1 + x_2 + 4x_3 = 9$ with $x^{\circ} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$8x_1 - 3x_2 + 2x_3 = 20$

$4x_1 + 11x_2 - x_3 = 3$

QUESTION TWO (20 MARKS)

(a) Consider the data in the table

x	0	6	20	45
y	30	48	88	238

Find:

(i) The Newton's divided difference interpolating polynomial. [4 marks]

(ii) The value of y at $x = 15$ [2 marks]

(b) The data in the table represents time (t) and the corresponding velocity (v) of a particle moving with non-uniform velocity

t	0.0	1.0	1.5	2.0
V	2.5	3.8	4.6	5.3

Use Lagrange's interpolation to determine the time when the velocity of the particle is 2.75. [5 marks]

(c) Find the eigenvalues and eigenvectors of the matrix A using the power method with $x^{\circ} = [1, 1, 1]^T$ [7 marks]

$$A = \begin{bmatrix} 6 & 3 & 1 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix}$$

QUESTION THREEE (20 MARKS)

(a) Use the weighted least square method to find a polynomial of degree 2 that fits the data below given that the weights on (1.1, 1.96), (1.5, 2.45) and (2.1, 3.18) are 0.4, 1.2 and 0.6 respectively. [5 marks]

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

(b) (i) Define the Chebyshev polynomial [2 marks]

(ii) Deduce the recurrence relation for the Chebyshev polynomial and use it to show that $T_3(x) = 4x^3 - 3x$ [7 marks]

(iii) Express $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$ in terms of Chebyshev polynomials [6 marks]

QUESTION FOUR (20 MARKS)

(a) Let $f(2,2) = 4, f(3,2) = 20$ and $f(2,3) = 15$ use triangular interpolation to approximate $f(2.5, 2.75)$ [4 marks]

(b) Consider the function $f(x) = e^x$ at $x = 0.1, x = 0.6, x = 1.0\ and\ x = 2.1$

(i) Use Newton's interpolating polynomials to estimate to $4.p.f(0.12\ and\ f(2.0)$ [7 marks]

(iii) Use stirlings formula to evaluate f(1.3) and find the percentage error in the approximation. [4 marks]

(c) Find the dominant eigenvalue and the corresponding eigenvector for matrix A to 3.s.f with $x^0 = [1,1,1]$ after 5 iterations $A = \begin{pmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{pmatrix}$ [5 marks]

QUESTION FIVE (20 MARKS)

(a) Find the interpolating polynomial passing through the following points (1,-6), (2, 2), (4, 12) and (3, -10) using

(i) Lagranges [5 marks]
(ii) Vandermonde [4 marks]

(b) (i) Explain the meaning of inverse interpolation [1 mark]

(ii) Let $y = x^3 - 2x^2 + 0.5$. Find one root of the equation that lies between 0.5 and 0.75 [6 marks]

(c) Use Hermites interpolation based on divided differences to approximate f(0.25) for the data

x_i	$f(x_i)$	$f'(x_i)$
0.1	-0.621	3.585
0.2	-0.284	3.140
0.3	+0.007	2.667
0.4	+0.284	2.165
