CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN MATHEMATICS

MATH 826: NUMERICAL ANALYSIS II

STREAMS: MSC (APP MATH) **TIME: 3 HOURS**

DAY/DATE: TUESDAY 10/12/2019 **2.30 PM – 5.30 PM**

INSTRUCTIONS:

ANSWER ANY THREE QUESTIONS

QUESTION ONE (20 MARKS)

(a) Consider, the initial value problem $y' = 3x^2y, y = (0) = 1$

(i) Find the exact solution of y when $x = 0.8$ [3 marks]

(ii) Solve using Euler's standard method with $h = 0.25$ [11 marks]

(b) Write down the Runge-Kutta methods for solving ODE of the form
$\frac{dy}{dx} = f(x, y); y(0) = y_0$ [6 marks]

(i) Order 2
(ii) Order 3
(iii) Order 4

QUESTION TWO (20 MARKS)

(a) Use Picards methods up to the second iteration corresponding to x=0.2 for the particular solution of $\frac{dy}{dx} = x + z, \frac{dz}{dx} = x - y^2$ given that when $x = 0, y = 2$ and $z =$1 [11 marks]

(b) Consider IVP $\frac{dy}{dx} = x^2 + y; \quad y(0) = 1$
Taking $h = 0.05$ approximate $y(0.1)$using Euler's modified method. [9 marks]

QUESTION THREE (20 MARKS)

(a) Consider the differential equation

$$\frac{d2y}{dt^2} + \frac{dy}{dt} + y = 1; \;\; y(0) = y'(0) = 0$$

(i) Use RK4 method to solve the IVP with h=0.2 and $0 \leq t \leq 0.2$

(ii) Approximate $y(0.2)$ and $z(0{,}2)$ [13 marks]

(b) Apply Picards method up to the third iteration to find the approximate solution to the D. E [7 marks]

$$\frac{dy}{dx} = x + y^2 \;; \;\; y(0) = 0$$

QUESTION FOUR (20 MARKS)

(a) Use Runge-Kutta method for order 4 with $h = 0.5$ for$0 \leq n \leq 1.5$ to solve IVP to 4 d.p [12 marks]

$$\frac{dy}{dt} = y - t^2 + 1; \; y(0) = 0.5$$

(b) Use Taylors series to solve the IVP and approximate $y(1.3)$ to $4\mathrm{d.p}$ [8 marks]

$y' = x + y \qquad y(2) = 0$
