CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

MATH 859: FINANCIAL TIME SERIES AND RISK MANAGEMENT

STREAMS: MSC (APPLIED STATISTICS) **TIME: 3 HOURS**

DAY/DATE: TUESDAY 06/08/2019 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS:

- **Attempt any THREE questions**

QUESTION 1 (20 MARKS)

(a) Asset volatility is one of the most important characteristics that is studied in the financial risk. However, it is not observable. Explain how it is usually measured by statisticians. (5 marks)

(b) Explain five characteristics of volatility. (5 marks)

(c) Consider the Autoregressive Conditional Heteroscedasticity model of order 1, ARCH (1) given as follows

$a_t = \sigma_t e_t$
$\sigma_t^2 = \sigma_0 + \alpha_1 a_{t-1}^2$

Where $e_t \sim N(0, \sigma), \alpha_0 > 0 \; and \; \alpha_1 \geq 0$. Required is to show that:

(i) $E(a_t) = 0$ (2 marks)

(ii) $Var\,(a_t) = \frac{\alpha 0}{(1-\alpha_1)}$ (3 marks)

(iii) Under normality assumptions, if a_t is fourth order stationary with $m_4 = E(a_t^4)$ then

$$m_4 = \frac{3\alpha_0^2(1+\alpha_1)}{1-\alpha_1)(1-3\alpha_1^2}$$ (5 marks)

QUESTION 2 (20 MARKS)

(a) Most financial studies involve returns, instead of prices of assets. Explain why. (5 marks)

(b) Consider a GARCH (p, q) model represented asan ARMA process given that $e_t^2 = \sigma_t^2 + v_t$ where $E_{t-1}(v_t) = 0$ and $v_t \in [-\sigma_t^2, \infty]$

$$e_t^2 = \alpha_0 + \sum_{j=1}^{\max(p,q)} (\alpha_j + \beta_j) e_{t-j}^2 + (v_t - \sum_{i=1}^{p} \beta_i v_{t-i})$$

$where\ e_t^2 \sim ARMA\ (m,p)\ with$
$m = \max(p,q). Show\ that\ E_t\ [\sigma_{t+k}^2] converges\ to\ \sigma^2 as\ k \to \infty$ (15 marks)

QUESTION 3 (20 MARKS)

(a) Explain in details the steps involved in building an ARCH model. (10 marks)

(b) Let η be a risk measure. It is said that η is coherent if for any two loss random variables X and Y it satisfies four conditions. Explain these four conditions very briefly using algebraic expressions. (4 marks)

(c) Explain the three methods used in the Value at Risk (VaR) computation. (6 marks)

QUESTION 4 (20 MARKS)

(a) All models are wrong, but some are useful (Box, 1976). In your opinion, do you agree or disagree with the author? Explain. (5 marks)

(b) Explain the steps involved in building a volatility model for an asset return series. (5 marks)

(c) Consider the ARCH (1) model in question 1c, show that

$$\frac{E(a_t^4)}{[Var\ (a_t]^2} = 3\frac{1-\alpha_1^2}{(1-3\alpha_1^2)} > 3$$ (10 marks)

QUESTION 5 (20 MARKS)

(a) In financial time series, there exist stylized facts about the return series. State and explain five stylized facts of return series data. (5 marks)

(b) Consider a Ksh. 1,000 million portfolio of medium-term bonds. Suppose the confidence interval is 95%, what is the maximum monthly loss under normal markets over any month? (Let for the 95% confidence interval, the lowest monthly log return (r_t)=1.7% (3 marks)

(c) Consider an investor had a Ksh. 10 million portfolio of bonds in a long position suppose the confidence interval is 95%. The actual daily standard deviation of the portfolio over

one trading year is 3.67%, what is the daily VaR of this portfolio? What is the VaR for a 1-month horizon (30 days)? Use 1.645, as the z-score for 95% confidence interval. (5 marks)

(d) Determine the Yule-Walker estimates for the GARCH (p, q) process. (7 marks)
