CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

MATH 921: NUMERICAL ANALYSIS I

STREAMS: PhD **TIME: 3 HOURS**

DAY/DATE: MONDAY 12/08/2019 **8.30 A.M. – 11.30 A.M.**

INSTRUCTIONS:

- **Answer ALL questions.**

QUESTION ONE

(a) Solve the non-linear system

$$3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0$$
$$x_1^2 = 81(x_1 + 0.1)^2 + \sin x_3 + 1.06 = 0$$
$$e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

When the initial approximation

$$X^0 = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.1 \end{bmatrix} \text{ upto } x^{(1)}$$

Using Newton's method (8 marks)

(b) Use Fibonacci algorithm to minimize

$$f(x) = \frac{-1}{(x-1)^2}\{\log x - 2(\frac{x-1}{x+1})\}$$

If it is known that the minimizer is in the range of [1.5, 4.5]. Reduce the interval to $(\frac{2}{21})$ of the original up to 5 iterations. (7 marks)

QUESTION TWO

(a) Minimize

$f(x_1x_2) = 4(x_1 - 5)^2 + 6(x_2 - 6)^2$. Use the method of Welder and Mead. The initial simplex has the following three vertices

A(8, 9), B(10, 11), C(8, 11) up to 4 iterations (15 marks)

QUESTION THREE

(a) Obtain the solution of the following by Crouts method

$4x + y - t = 13$
$3x + 5y + 2t = 21$
$2x + y + 6t = 14$ (5 marks)

(b) Determine the root of
$x^4 + x^3 - x + 5 = 0$ which has between 2 and 3, correct to 3 dp. By Newton-Ralpson's method. (5 marks)

(c) Compute the real root of $xlog\, x - 1.2 = 0$ by NR methods given that the real root has between 2 and 3. (5 marks)

QUESTION FOUR

(a) Find the maximum of

$A = (h + b + 10)(\frac{2{,}272{,}000}{hb} + 2b + 5)$ by Newton's method given that

$a = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $G = \begin{bmatrix} 49.92 \\ 375.1 \end{bmatrix}$

$J = \begin{bmatrix} 4.998, & 2.227 \\ 2.227, & 8.998 \end{bmatrix}$ (8 marks)

(b) Find the minimum of

$f(x, y) = x^4 + y^4 + (2x + y - 5)^2$ by DFP method starting at (0,0) using 3 iterations. (7 marks)
