CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

MATH 205: ELEMENTS OF SET THEORY

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 14/12/2017 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

- **Answer question one and any other two questions.**
- **Do not write on the question paper**

1. (a) List the elements of the following sets where $\mathbb{N} = \{1,2,3,\ldots.\}$

(i) $A = \{x:x \epsilon \mathbb{N}, x \text{ is even}, x < 15\}$

(ii) $B = \{x:x \epsilon \mathbb{N}, 4 + x = 3\}$ [2marks]

(b) Given that $U = \{1,2,3,\ldots.,9\}$, $A = \{1,2,3,4,5\}$, $B = \{5,6,7,8,9\}$ and $C = \{4,5,6,7\}$ find

(i) $A \oplus B$. [2marks]

(ii) $A^c \cap B^c$ [2marks]

(c) Prove that $A \cup B = B \cup A$. [2marks]

(d) Determine the number of elements in the power set of A = {days of the week}. [2marks]

(e) Define a well ordered set. [2marks]

(f) Given that $f(x) = x^2$ and $g(x) = x + 3$, find ;

(i) $(fog)(x)$ [2marks]

(ii) (gof)(x) [2marks]

(g) If $\mathcal{A} = \{\{1,2,3,4\},\{2,3,4,5\},\{3,4,5,6\},\{3,4,7,8,9\}\}$, find ;

(i) $\cup \mathcal{A}$

(ii) $\cap \mathcal{A}$ [4marks]

(h) Prove that $[0,1] \approx (0,1)$ [4marks]

(i) Consider the Hasse diagram of an ordered set A below;

Find the ,

(i)minimal and maximum elements of A. [2marks]

(ii) First and last elements of A . [2marks]

2. (a) Study the ordered set A represented below;

For each $a \in A$, let p(a) denote the set of predecessors of a, that is, $p(a) = \{ x : x \leq a \}$. Let P(A) the collection of all predecessor sets of A and let P(A) be an ordered set by inclusion. Draw the Hasse diagram for P(A) . [8marks]

(b) Suppose A and B are ordered sets. Show that the product order on A x B defined by $(a,b) \preccurlyeq (c,d)$ if $a \leq b$ and $b \leq d$ is a partial ordering on A x B. [10marks]

(c) Let S = {a,b,c,d,e,f,g} be ordered as in the figure below and lrt X = {c,d,e}

Find the upper and lower bounds of X. [2marks]

3. (a) Let λ be any ordinal number. Prove that λ +1 is the immediate successor of λ. [5marks]

(b) Prove that if A and B are well ordered sets and:

S = {x:x ∈ A,s(x) ≈ s(y) where y∈ B}

T = {y:y ∈ B,s(y) ≈ s(x) where x∈ A}

Then S is similar to T. [10marks]

(c) Given that g(x) = $\frac{2x-3}{5x-7}$, find g^{-1}(x) . [5marks]

4. (a) Prove that $(U_i A_i)^c = \cap_i A_i^c$. [5marks]

(b) In a survey of 60 people, it was found that ; 25 read the daily nation, 26 read standard , 26 read the people, 9 read both the nation and the people, 11 read both the nation and standard, 8 read the standard and the people and 3 read all the three newspapers.

(i) Find the number of people who read at least one of the newspapers. [3marks]

(ii) With N,S and P representing the people who read the nation, standard and people respectively draw a venn diagram with its 8 regions filled. [4marks]

(iii) Find the number of people who read exactly one newspaper. [2marks]

(c) Prove that a countable union of finite sets is countable. [6marks]

5. (a) For each m ∈ P, let A_m be the subset of P given by

A_m={m,2m,3m,….} = {multiples of m} find ;

(i) $A_2 \cap A_7$ [2marks]

(ii) $A_3 \cap A_{12}$ [2marks

(b) state the axiom of choice . [2marks]

(c) Find if the sequence x_n = $\{1 + (-1)^n \frac{1}{n}$: n∈N} is bounded or not .If bounded state its supremum and infimum . [4marks]

(d) Prove that A∪(B∩ C) = (A∪B) ∩(A∪C). [5marks]

(e) Given that f(x) = 2x +1 and g(x) = x^2, show that $(f^{-1} \circ g^{-1}(1)$ = $(g \circ f)^{-1}(1)$. [5marks]
