

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

**EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE
(ECONOMICS & STATISTICS) AND BACHELOR OF EDUCATION (ARTS
& SCIENCE)**

MATH 326: NUMERICAL ANALYSIS

STREAMS: BSC(ECOSTAS) BED(ART&SCI)

TIME: 2 HOURS

DAY/DATE: THURSDAY 14/12/2017

8.30 A.M – 10.30 A.M

INSTRUCTIONS:

1. (a) If $y(0) = 1$, $y(2) = 1$ and $y(3) = 10$, find the polynomial associated with this data using Lagrange's polynomial and hence evaluate $f(1.5)$. [5marks]
- (b) Use Newton Raphson method to find the root of $x^3 - 6x + 4 = 0$ between 0 and 1 to 5 decimal places. 4 iterations only. [4marks]
- (c) Approximately $\int_0^2 \sqrt{x} dx$ with $n = 4$ to 5 decimal places using the Simpson's $\frac{1}{3}$ rule. [4marks]
- (d) Apply the determinant form to find the linear approximating formula and use to estimate $f(9.7)$. [4marks]

x	9.5	11.0
y	2.2513	2.3979

- (e) Briefly explain why polynomials are chosen to approximate functions. [3marks]
- (f) Solve the system using Cramer's rule.

$$6a + 10.5b = 9.80$$

$$10.5a + 22.75b = 21.945 \quad [5\text{marks}]$$

(g) If $P = \frac{6x^2y}{z^3}$ and $\Delta x = 1.0 \times 10^{-3}$, $\Delta y = 2.0 \times 10^{-3}$ and $\Delta z = 100 \times 10^{-3}$ compute the relative error in p given that $x = z = 1$ and $y = 2$. [5marks]

2. (a) Consider the values in the table below.

X	0	1	2	3
Y	0.00	0.84	1.82	0.42

- (i) Construct the table of divided differences. [3marks]
- (ii) Use Newton's divided difference interpolation formula to obtain the polynomial for the given data based at x_0 .
- (iii) Compute $f(x)$ at $x = 1.4$.
- (iv) Use the secant method to find the real roots of the equation $x = \cos x$ to 4 decimal places taking $x_0 = 0.5$ and $x_1 = 1$ with 4 iterations. [6marks]

(c) Solve the system of linear equations using Gauss Jordan's method of row reductions. [5marks]

$$x - y + z = -3.5$$

$$2x + 3y + 2z = 8$$

$$2x - 2y + 4z = -12$$

3. (a) Derive the Newton Raphson's formula for evaluating $\sqrt[3]{N}$ where N is a positive number and hence find $\sqrt{5}$ using 5 iterations. [8marks]

(b) Compute and interpret the condition number for $f(x) = \tan x$ at $a = 1.7$. [5marks]

(c) Consider the data in table below and use it to interpolate the value of y where $x = 1.91$ using the Newton's forward interpolation formula (NFIF) [6marks]

X	1.7	1.8	1.9	2	2.1	2.2
F(x)	5.4739	6.0496	6.6859	7.3891	8.1662	9.025

4. (a) Compute a root of $x + \log x - 2 = 0$ which lies between 1 and 2 to one decimal places using the Bisection method with 6 iterations. [7marks]

(b) Determine the maximum relative error for the function

$F = 3x^3y^2 + 5y^2z^2 - 7x^2z^2 + 38$ for $x = y = z = 1$ and $\Delta x = -0.05$, $\Delta y = 0.001$ and $\Delta z = 0.02$. [8marks]

(c) Consider the values in the table below

X	0.02	0.4	0.6	0.8
$F(x) = x^4$	0.0016	0.0256	0.1296	0.4096

(i) Write down the four point differentiation formula. [1mark]

(ii) Use the four point formula to estimate $f'(0.2)$ and $f''(0.4)$. [4marks]

5. (a) Show that $x^3 - 7x + 14x - 6 = 0$ has a root in the interval $[1,2]$ and hence find the root using the regula falsi method with 5 iterations. [6marks]

(b) Compute Lagrange's cubic interpolation for the data in the table. [4marks]

X	1	2	3	4
F(x)	3	9	11	18

(ii) Use the polynomial in b(i) above to evaluate $f(1.6)$ and $f(3.2)$. [3marks]

(c) A rod is rotating in a plane. The table gives the degree in radians through which the rod has turned.

t	0	0.2	0.4	0.8	1.0	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

Using Newton's backwards interpolation formula (NBIF) find θ when $t = 0.9$. [7marks]