

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 803: ADVANCED MATHEMATICS FOR CHEMISTS

STREAMS: MSC (APPLIED MATHS)

TIME: 3 HOURS

DAY/DATE: MONDAY 06/04/2020

11.30 A.M. – 2.30 P.M.

INSTRUCTIONS: Answer any FOUR questions

QUESTION ONE (15 MARKS)

- (a) Find the domain of the function below $f(x) = \sqrt{3x^2 - 3x - 18}$ [2 marks]
- (b) Given that $y = \cos^{-1}(2x + 3)$, find $\frac{dy}{dx}$ [2 marks]
- (c) Evaluate $\lim_{x \rightarrow \infty} \frac{6x^3+5}{2x^3+1}$ [5 marks]
- (d) Given that $y = f(x) = \frac{1}{x^2+1}$, find $\frac{dy}{dx}$ from first principles [3 marks]
- (e) Differentiate $y = 2^{3x}$ at the point $x = 1$ [2 marks]
- (f) Find the equations of lines tangent to the curve $y^2 - 6x^2 + 4y + 1 = 0$ at the point $(2, 1)$ [4 marks]

QUESTION TWO (15 MARKS)

- (a) Determine $\int \frac{4}{(5x-3)} dx$ [2 marks]
- (b) Evaluate $\int_0^1 5xe^{4x} dx$ using the by parts method [2 marks]
- (c) Determine the volume of the solid of revolution formed when the curve $y = 2$ rotated 360° about the x axis between the limits $x = 0$ to $x = 3$ [3 marks]
- (d) Determine $\int \frac{11-3x}{x^2+2x-3} dx$ using partial fractions [4 marks]

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(e) Evaluate the following integrals

(i) $\int \ln 10 + \frac{1}{x^2} - \sqrt{x^3} dx$ [2 marks]

(ii) $\int \frac{dx}{16x^2+9}$ [2 marks]

QUESTION THREE (15 MARKS)

(a) Find the volume of revolution bounded by the region $y = x^3, x = 2$ and $x = 4$ about $y = 1$ [3 marks]

(b) Find the area of the triangle PQR with vertices $p = (3 \ 4 \ 7), Q = (0 \ 6 \ 1)$ and $R = (5 \ -2 \ 4)$ [2 marks]

(c) Verify whether the vectors $(-1 \ 2 \ 2), (2 \ -3 \ 1), (-4 \ 7 \ 3)$ are coplanar [2 marks]

(d) Solve the differential equation $y'' + 4y' + 5y = 0$ [2 marks]

(e) State the order, linearity and degree of the following differential equations.

(i) $\frac{d^3y}{dx^3} + 4e^y = 0$ [2 marks]

(ii) $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = \left(\frac{d^3y}{dx^3}\right)^3$ [2 marks]

(f) Determine whether $y = \frac{x^2}{3} + \frac{1}{x}$ is a solution of the differential equation $x \frac{dy}{dx} + y = x^2$ [2 marks]

QUESTION FOUR (15 MARKS)

(a) Solve the differential equation $y' = x(1 + y^2)$ by the method of separation of variables [3 marks]

(b) Verify whether the differential equation below is exact hence solve $(4x^3y^3 + 3x^2)dx + (3x^4y^2 + 6y^2)dy = 0$ [3 marks]

(c) Find an integrating factor for $(3xy + 6y^2)dx + (2x^2 + 9xy)dy = 0$ and solve the equation [3 marks]

(d) Find the general solution of

(i) $y'' + 6y' + 5y = 0$ [3 marks]

(ii) $y'' + 4y' + 13y = 0$ [3 marks]

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QUESTION FIVE (15 MARKS)

- (a) Find the angle between the vectors $A = 3i + 6j + 9k, B = -2i + 3j + k$ [2 marks]
- (b) Find the area of the parallelogram formed by the vectors
 $A = 3i + j - 2k, B = i - 3j - 2k$ [2 marks]
- (c) Find the value of c for which the vectors $ci + j + k$ and $-i + 2k$ are perpendicular [2 marks]
- (d) Find the inverse of matrix $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$ [4 marks]
- (e) Solve the simultaneous equations below both by the crammers rule and the matrix cofactor method [5 marks]
- (i) $3x + y + z = 10$
- (ii) $2x - 3y + 5z + 9 = 0$
- (iii) $x + 2y - z = 6$
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