CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTERS OF SCIENCE IN PURE MATHEMATICS

MATH 807: GROUP THEORY I

STREAMS: MSC (PURE MATH) P/T TIME: 3 HOURS

DAY/DATE: TUESDAY 14/04/2020 8.30 AM – 11.30 AM

INSTRUCTIONS:

- Answer any three questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- This is a **closed book exam,** No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (20 MARKS)

(a) Define a subnormal series of a group G and find a subnormal series of $\mathbb{Z}_{30}$. [3 marks]

(b) Find a composition series of the dihedral group D_4. [2 marks]

(c) State without proof Jordan-Holder theorem. Find all the composition series of $\mathbb{Z}_{15}$ and verify Jordan-Holder theorem. [7 marks]

(d) Define a composition series and prove that every finite group has a composition series. [5 marks]

(e) Show that every finite abelian group is nilpotent. [3 marks]

QUESTION TWO (20 MARKS)

(a) Let G be a group and G^I be the derived group of G. show that $G/_{G^I}$ is abelian and that $G/_{H}$ if abelian then $G^I \subseteq H$. [7 marks]

(b) Show that G^I is a normal subgroup of G. [4 marks]

(c) If G is a group $G^I \subset H \subset G$, show that $H \lhd G$. [4 marks]

(d) Show that every p-group is nilpotent. [5 marks]

QUESTION THREE (20 MARKS)

(a) Show that S_3 is a semi direct product of subgroup isomorphic to C_3 by a subgroup isomorphic to C_2. [3 marks]

(b) Define a soluble group and show whether or not S_3 is soluble. [4 marks]

(c) Let $H \lhd G$. Show that if both H and $G/_H$ are soluble, then G is soluble. [4 marks]

(d) Show that Hxl and IxK are normal subgroups of HxK: these two subgroups generate HxK and their intersection is (1,1). [6 marks]

(e) List all the abelian and non-isomorphic abelian groups of order 720. [3 marks]

QUESTION FOUR (20 MARKS)

(a) Let G be a group

(i) Define an upper central series of G. [2 marks]

(ii) Define a lower central series of G. [2 marks]

(b) Define a nilpotent group and show that a nilpotent group is soluble. [5 marks]

(c) Show that a group G is soluble if and only if its derived series terminates at the identity group. [5 marks]

(d) (i) Give the subgroup structure of S_3. [4 marks]

(ii) List all the sylow 2-subgroups and all the sylow 3-0subgroups of S_3. [2 marks]
