

CHUKA



UNIVERSITY

## UNIVERSITY EXAMINATIONS

**EXAMINATION FOR THE AWARD DEGREE OF  
BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)**

**ACMT 202: FUNDAMENTAL OF ACTUARIAL MATHEMATICS II**

STREAMS:

TIME: 2 HOURS

DAY/DATE: THURSDAY 9/04/2020

11.30 A.M - 1.30 A.M.

**INSTRUCTIONS**

Answer questions ONE (compulsory) and any other TWO questions

**QUESTION ONE (30 MARKS)**

- a) A life aged 50 who is subject to the mortality of the English Life Tables No. 15 (Females) effects a pure endowment policy with a term of 20 years for a sum assured of Kshs10, 000.
- I. Write down the present value of the benefits under this contract, regarded as a random variable. (2 Marks)
  - II. Assuming an effective rate of interest of 10% per annum, calculate the mean and the variance of the present value of the benefits available under this contract. (4 Marks)
- b) The following is an extract from a life table with a select period of 1 year.

Age $x$	$(x)$ $l[x]$	$x + 1$ $lx+1$	$age\ x + 1$ $agex + 1$
55	90,636	90,032	56
56	89,739	89,132	57
		88,151	58
		87,094	58
		85,874	60
		84,586	61

Evaluate  $a_{56:5}$  and  $a_{[56]:5}$  at 5 % per annum interest.

(6 Marks)

- c) A life aged 40 effects a 25-year without profits endowment assurance policy with a sum assured of Kshs50,000 (payable at the end of the year of death or on survival to the end of the term). Level premiums are payable annually in advance throughout the term of the policy or until earlier death of the life assured. Calculate the level premium, P, using the following premium basis

Mortality: English Life Tables No. 15  
Interest: 6% p.a. (5 Marks)

- d) State four reasons why life office need reserves (5 Marks)
- e) Calculate values for the following functions, assuming AM92 mortality:

I.  $\frac{D_{50}}{D_{40}} a_{50}$  at a rate of interest of 6% (3 Marks)

II.  $A_{\{31\}:29}$  at a rate of interest of 4% (1 Marks)

- f) The payments under a special deferred annuity are payable continuously from age 60 and increase continuously at the rate of 5% *pa* compound. The payment stream starts at the rate of Kshs200 *pa*. Assuming AM92 select mortality before age 60 and PFA92C20 mortality after age 60, calculate the value of the annuity for a female life now aged 40, if interest is 5%. (4 Marks)

## QUESTION TWO (20 MARKS)

- a) Show that  $A_x = vq_x + vp_x A_{x+1}$  (5 Marks)
- a) Given that  $p_{60} = 0.985$ ;  $p_{61} = 0.98$ ;  $i = 0.05$  and  $A_{62} = 0.6$ , evaluate  $A_{61}$  and  $A_{60}$  (5 Marks)
- a) A life aged exactly 60 wishes to arrange for a payment to be made to a charity in 10 years' time. If he is still alive at that date the payment will be Kshs1000. If he dies before the payment date, the amount given will be Kshs500. Assuming an effective interest rate of 6% per annum and mortality according to ELT No.15- Males, calculate the standard deviation of the present value of the liability.

(10 Marks)

**QUESTION THREE (20 MARKS)**

A whole life annuity is payable continuously to a life now aged 60. The rate of payment at time  $t$  is:

$$r(t) = 10,000(1.02)^t \quad (t > 0)$$

- a) Write down an expression for the present value of the annuity in terms of annuities-certain. (4 Marks)
- b) Write down expressions for the expected present value and variance of the present value of the annuity. (4 Marks)
- c) Calculate the expected value and the variance of the annuity assuming AM92Ultimate mortality and 6.08% *pa* interest. (4 Marks)
- d) Simplify your expressions for the present value and its expectation assuming that  $i = 0.02$ . (4 Marks)
- e) Calculate the expected present value of the annuity assuming AM92 Ultimate mortality and 2% *pa* interest. (4 Marks)

**QUESTION FOUR (20 MARKS)**

An insurer issues a combined term assurance and annuity contract to a life aged 35. Level premiums are payable monthly in advance for a maximum of 30 years. On death before age 65 a benefit is paid immediately. The benefit is Ksh 200,000 on death in the first year of the contract, Kshs195,000 on death in the second year, Kshs190,000 on death in the third year, *etc*, with the benefit decreasing by Kshs5,000 each year until age 65. No benefit is payable on death after age 65. On attaining age 65 the life receives an annuity of Kshs10,000*pa* payable monthly in arrears.

Calculate the monthly premium on the basis of:

Mortality: up to age 65: AM92 Select  
 over age 65: PFA92C20  
 Interest: 4% *pa*  
 Expenses: none

(20 Marks)

**QUESTION FIVE (20 MARKS)**

a) An annuity is payable continuously throughout the lifetime of a person now aged exactly 60, but for at most 10 years. The rate of payment at all times  $t$  during the first 5 years is Kshs10,000 $pa$ , and thereafter it is Kshs12,000  $pa$ . The force of mortality of this life is 0.03  $pa$  between the ages of 60 and 65, and 0.04  $pa$  between the ages of 65 and 70. Calculate the expected present value of this annuity assuming a force of interest of 0.05  $pa$ . (10 Marks)

b) Assuming that mortality and interest are as in (a) above, calculate the expected present value of:

- I. a 10-year term assurance issued to the life in (a), which pays Kshs50,000 immediately on death
- II. a 10-year endowment assurance issued to the life in (a), which pays Kshs50,000 on maturity or immediately on earlier death.

(10 Marks)

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